

# Political Economy

Pierre Boyer

École polytechnique - CREST

Fall 2022

# Limits of The Downsian Model

- The candidates are perfectly informed about the preferences of the voters (or at least they know the preferred policy of the median voter).
- The framework is essentially limited to one-dimensional policy spaces.
- Political parties are exogenously given and candidates do not self-select.
- Candidates can commit to applying their campaign promises.
- Platforms are non-ambiguous.

1. **Probabilistic Voting: Random Utility Model**
  - a. Office seeking candidates
  - b. Ideological Candidates
2. Probabilistic Voting: Idiosyncratic Biases
3. Probabilistic Voting and Redistribution
4. Citizen Candidate Models

# Random Utility Model

- Suppose the preferences of the voters satisfy either
  - ⓐ Single Peakness
  - ⓑ Single-crossing property and the median voter's preferences are single-peaked.
- Let  $\pi_m$  be the favorite policy of the median voter.
- To capture uncertainty about preferences, we assume that the candidates do not observe  $\pi_m$  but instead believe that  $\pi_m \sim F(\cdot)$ .
- We assume that the beliefs of the candidates coincide: polls are public information.

# Double Median

- Let  $\pi_m^* = F^{-1}(1/2)$  be the median (relative to the uncertainty) favorite policy of the median voter.

## Theorem (Convergence)

*If the two candidates are office seeking, there exists a unique pure strategy Nash equilibrium in which both candidates choose the same platform  $\pi_m^*$ .*

- For simplicity, we assume that  $F(\cdot)$  has full support.
- Suppose the candidates announce  $\pi_L < \pi_R$ .
- $\pi_L$  ( $\pi_R$ ) wins for all the profiles with  $\pi_m < \pi_L$  ( $\pi_m > \pi_R$ )
- Look at the deviation  $\pi_L \rightarrow \pi'_L = \pi_L + \frac{2}{3}(\pi_R - \pi_L)$
- Then for all the profiles with  $\pi_m < \pi'_L$ ,  $\pi'_L$  is a winner

- Two cases
  - ⓐ If for some of these profiles  $\pi_L$  was losing, then the deviation is profitable.
  - ⓑ If  $\pi_L$  was already winning for all of these profiles, then  $\pi_R \rightarrow \pi'_R = \pi_R - \frac{2}{3}(\pi_R - \pi_L)$  is a profitable deviation from  $\pi_R$ .
- Hence  $\pi^L = \pi^R$ .
- But then if  $\pi_L = \pi_R \neq \pi_{m'}^*$ , any candidate can move  $\epsilon$  closer to  $\pi_m^*$  and win with a probability greater than  $F(\pi) > 1/2$ .

# Ideological Candidates: an Example

- $\Pi$  is an interval of  $\mathbb{R}$  that contains  $[-1, 1]$ .
- All voters have symmetric single-peaked preferences
- $\pi_m$  is uniformly distributed on  $[-\delta, \delta]$ .
- Candidates  $l$  and  $r$  with preferences

$$u_l(\pi) = -(\pi + 1)^2,$$

$$u_r(\pi) = -(\pi - 1)^2$$

- $l$  wins if  $\pi_m < \frac{\pi_\ell + \pi_r}{2}$



# Equilibrium: Partial Convergence

- In equilibrium, it must be true that  $\frac{\pi_\ell + \pi_r}{2} \in [-\delta, \delta]$ .

- $\ell$ 's payoff

$$-(\pi_\ell + 1)^2 \frac{1}{2\delta} \left( \frac{\pi_\ell + \pi_r}{2} + \delta \right) - (\pi_r + 1)^2 \frac{1}{2\delta} \left( \delta - \frac{\pi_\ell + \pi_r}{2} \right)$$

- By symmetry  $\pi_\ell^* = -\pi_r^*$ .
- Then, plugging  $\pi_\ell^* = -\pi_r^*$  in the FOC:  $\pi_\ell^* = -\frac{\delta}{1+\delta} = -\pi_r^*$ .
- Hence there is partial convergence, and  $\pi_\ell^*, \pi_r^* \rightarrow \pi_m = 0$  as  $\delta \rightarrow 0$ .

1. Probabilistic Voting: Random Utility Model
  - a. Office seeking candidates
  - b. Ideological Candidates
2. **Probabilistic Voting: Idiosyncratic Biases**
3. Probabilistic Voting and Redistribution
4. Citizen Candidate Models

# Probabilistic Voting

- Hinich (1977), Coughlin and Nitzan (1981), Ledyard (1981,1984), Banks and Duggan (2005) for a unifying framework.
- The policy space  $\Pi \subseteq \mathbb{R}^n$  (multidimensional).
- There are two exogenously given, office seeking candidates  $A$  and  $B$  who choose policies  $\pi_A$  and  $\pi_B$  to maximize their vote share.
  - ▶ For example, because a higher share of the votes leads to more post-election power (whether they win or they lose), or more opportunities to earn rents, or increased campaign funding in future elections.

# Voters Preferences

- The preferences of voter  $i$  over  $\Pi$  are given by  $u_i(\pi)$ .
- Voters also care about other dimensions that the candidates cannot control (image, personality, likeability, perceived policy positions on issues that are left out of the debate...)
- The payoff of  $i$  if candidate  $k$  is elected is given by

$$U_i(k) = u_i(\pi_k) + \varepsilon_i^k$$

- The likeability factors  $\varepsilon_i^k$  are random from the point of view of the candidates

$$\varepsilon_i^B - \varepsilon_i^A \sim F_i(\cdot)$$

# Vote Shares

- Regularity assumptions:  $u_i(\cdot)$  and  $F_i(\cdot)$  continuously differentiable, the density of  $F_i(\cdot)$  is  $f_i(\cdot)$ .

$$\begin{aligned}\Pr(i \text{ votes for } A) &= \Pr(U_i(A) - U_i(B) > 0) \\ &= F_i(u_i(\pi_A) - u_i(\pi_B))\end{aligned}$$

- Candidate  $A$ 's expected number of votes is:

$$V_A(\pi_A, \pi_B) = \sum_{i \in \mathcal{I}} F_i(u_i(\pi_A) - u_i(\pi_B))$$

- $B$ 's expected number of votes is:

$$V_B(\pi_A, \pi_B) = \#\mathcal{I} - \sum_{i \in \mathcal{I}} F_i(u_i(\pi_A) - u_i(\pi_B))$$

# Convergence

- Assumptions (see Banks and Duggan, 2005. In Social Choice and Strategic Decisions.)
  - ①  $\Pi$  is compact and convex
  - ②  $u_i(\cdot)$  is concave.
  - ③  $V_A(\pi_A, \pi_B)$  is strictly concave in  $\pi_A$  and strictly convex in  $\pi_B$ .

# Convergence

- Under these assumptions, there exists a unique Nash equilibrium in which both candidates choose the same platform  $\pi^*$ .
- Furthermore  $\pi^*$  maximizes the following weighted social welfare function

$$\pi^* = \arg \max_{\pi} \sum_i f_i(0) u_i(\pi),$$

in which  $i$ 's weight is  $f_i(0)$ .

- In a Nash equilibrium

$$\pi_A^* \in \arg \max_{\pi} \sum_{i \in \mathcal{I}} F_i (u_i(\pi) - u_i(\pi_B^*)),$$

$$\pi_B^* \in \arg \max_{\pi} - \sum_{i \in \mathcal{I}} F_i (u_i(\pi_A^*) - u_i(\pi)).$$

- First Order Conditions for a Nash equilibrium with convergence

$$(\pi_A^* = \pi_B^* = \pi^*)$$

$$\sum_{i \in \mathcal{I}} f_i(0) u_i'(\pi^*) = 0, \forall i.$$

- This condition is the same as the FOC for the program

$$\max_{\pi} \sum_i f_i(0) u_i(\pi)$$



# Interpreting the Result

- Suppose a voter's utility is given by

$$U_i(C) = u_i(\pi_C) + \alpha_i \varepsilon^C,$$

where

- $\varepsilon^B - \varepsilon^A \sim F(\cdot)$  common to all voters.
  - $\alpha_i > 0$  is the weight of the bias for voter  $i$ .
- Then the weight of voter  $i$  is  $\frac{f(0)}{\alpha_i}$ .

# Interpreting the Result

- **Intuition:** the voters who care more about policy and less about the idiosyncratic factor have more weight in the political outcome.
  - Relevance of the “Less Ideological” (or “Swing”) voters: they are easier to “convince” through policy
- ⇒ “Swing voters” decide
- Note that the political outcome is ex ante Pareto efficient whenever  $E [\varepsilon_i^A - \varepsilon_i^B] = 0$ .
- It is also Pareto efficient if we decide that idiosyncratic biases should be ignored.

1. Probabilistic Voting: Random Utility Model
  - a. Office seeking candidates
  - b. Ideological Candidates
2. Probabilistic Voting: Idiosyncratic Biases
3. **Probabilistic Voting and Redistribution**
4. Citizen Candidate Models

# Probabilistic Voting and Redistribution

- Lindbeck and Weibull (1987)
- Voters  $\mathcal{I} = \{1, \dots, I\}$ , with incomes  $y_i > 0$ .
- A redistribution policy  $t \in \mathbb{R}^I$  is feasible and budget balanced if it satisfies:
  - Ⓐ  $y_i + t_i > 0$
  - Ⓑ  $\sum_{i \in \mathcal{I}} t_i = 0$
- Preferences with  $v'_i(\cdot) > 0$  and  $v''_i(\cdot) < 0$  and  $v'_i(0) = \infty$

$$u_i(t, C) = v_i(y_i + t_i) + \varepsilon_i^C$$

- $\varepsilon_i^B - \varepsilon_i^A \sim F_i(\cdot)$

# Equilibrium

- Nash equilibrium

$$t^A \in \arg \max_t \sum_{i \in \mathcal{I}} F_i \left( v_i(y_i + t_i) - v_i(y_i + t_i^B) \right) \quad \text{s.t.} \quad \sum_i t_i = 0,$$

$$t^B \in \arg \max_t \sum_{i \in \mathcal{I}} -F_i \left( v_i(y_i + t_i^A) - v_i(y_i + t_i) \right) \quad \text{s.t.} \quad \sum_i t_i = 0,$$

- Let  $\lambda^A$  and  $\lambda^B$  be the Lagrange multipliers associated with the constraint in each program. Then for every  $i$  and every  $C \in \{A, B\}$

$$v'_i(y_i + t_i^C) f_i \left( v_i(y_i + t_i^A) - v_i(y_i + t_i^B) \right) = \lambda^C.$$

# Policy Convergence

- Hence the following ratio is constant across individuals  $i$

$$\frac{v'_i(y_i + t_i^A)}{v'_i(y_i + t_i^B)}$$

- Suppose  $t^A \neq t^B$ , then there exists  $j$  such that  $t_j^A > t_j^B$ .
- But then it is true for every  $i$  that  $t_i^A > t_i^B$ .
- But then  $\sum_i t_i^A > \sum_i t_i^B$
- $t^A$  and  $t^B$  cannot both be budget balanced!

# Conclusions

- Hence in any Nash equilibrium in pure strategies, there is policy convergence  $t^A = t^B = t^*$ .
- Then there exists a constant  $\lambda > 0$  such that
$$\sum_{i \in \mathcal{I}} v'_i(y_i + t_i^*) f_i(0) = \lambda.$$
- And  $t^* = \arg \max_t \sum_i f_i(0) v_i(y_i + t_i)$  s.t.  $\sum_i t_i = 0$ .
- The voters who receive higher transfers are those with higher  $f_i(0)$ . As before, citizens who care less about the idiosyncratic factor receive more.

# Remarks on Probabilistic Voting

- The clear advantage of probabilistic voting models is that they allow to deal with multi-dimensional policy space.
- They also introduce some uncertainty in elections which seems realistic.
- The electoral game always admits a mixed strategy equilibrium.
- However, to ensure existence of a pure strategy equilibrium (or uniqueness, or policy convergence), we need to make assumptions that imply that the amount of uncertainty/randomness is substantial. Put differently, voters must care relatively little about the policy in question.



1. Probabilistic Voting: Random Utility Model
  - a. Office seeking candidates
  - b. Ideological Candidates
2. Probabilistic Voting: Idiosyncratic Biases
3. Probabilistic Voting and Redistribution
4. **Citizen Candidate Models**

# Citizen-Candidate Approach

- Once we recognize that candidates have policy preferences, we must address the question of what determines these preferences.
- Thus, in the model with ideological candidates what determines the  $+1$  and  $-1$ ?
- To understand this, we have to consider the decision of citizens of whether to run for office.
- This also raises the question of the number of candidates who decide to run.
- The **citizen-candidate model** seeks to explain the number of candidates running and their policy preferences.

- The **citizen-candidate model** offers a very different vision of candidate behavior.
  - Key assumption: no commitment
- ⇒ Candidates must run on their true ideologies.
- There are two distinct justifications for this assumption.
1. Candidates may prefer to be honest and therefore find it costly to misrepresent their true beliefs.
  2. Once elected, candidates' behavior may plausibly be driven more by their true ideologies than the ideologies they announce in the campaign. If this is the case, voters will recognize that what matters for predicting policy choices will be what the candidate truly believes rather than what he announces.

- The assumption that candidates must run on their true ideologies is fundamentally different than that prior models which assume candidates can reposition themselves at will.
- Casual empiricism seems to suggest that there is some truth to both positions.
- Empirical work – Lee, Morretti, and Butler (2004, *QJE*).

In support of the citizen-candidate model, it does not seem to be the case that legislators change their positions (as measured by voting records) when their constituency changes (say, via redistricting).

Moreover, survey evidence suggests that repositioning leads voters to distrust candidates' announced positions and their integrity.

- The citizen-candidate approach models elections as a three stage game.
- In Stage 1, citizens decide whether or not to enter the race as a candidate.
- Running entails a sunk cost which may be thought of as the time devoted to running a campaign.
- In Stage 2, citizens vote over the set of self-declared candidates.
- Voting is assumed not to be costly, so everybody votes.
- Voting is strategic in some treatments of the model (Besley and Coate) and sincere in others (Osborne and Slivinski).
- In Stage 3, the candidate with the most votes is elected (plurality rule) and follows his/her true ideology when in office.
- If there is a tie, the winning candidate is determined by the toss of a fair coin.

- When voting in Stage 2, citizens rationally anticipate that the winning candidate will follow his/her true ideology and this determines their payoffs from the different candidates.
- Importantly, citizens are assumed to know each candidate's true ideology.
- In Stage 1, candidates are assumed to perfectly anticipate how citizens will vote for any given set of candidates.

# Citizen-Candidate Approach

- The advantage of the citizen-candidate model is that it endogenizes the number of candidates and their policy positions.
- The disadvantage of the model is that it does not yield a unique prediction.
- There are many different possible equilibria, some involving spoiler candidates who run just to prevent other candidates from winning.

# Citizen Candidates

- Besley and Coate (1997), Osborne and Slivinski (1996)
- Idea: candidates are citizens who decide to step up to defend their ideal policy.
- Candidacy is endogenous.
- There is no commitment.



# Model

- $\mathcal{I} = \{1, \dots, I\}$
- $\Pi_i$  is the set of policies available to  $i$  (citizens may have different policy-making competence).

$$\Pi = \bigcup_{i \in \mathcal{I}} \Pi_i$$

- Utility  $U_i(\pi, j)$ , where  $j \in \mathcal{I} \cup \{0\}$  (ego-rents, likeability...)
- Every citizen can decide to run at cost  $\delta$ .
- Elections: the candidate with the highest number of votes wins, (uniform probabilities in case of a tie).
- If no one runs, the default policy is  $\pi_0 \in \bigcap_{i \in \mathcal{I}} \Pi_i$

1. Candidates declare themselves.
2. Citizens decide for whom to vote among the declared candidates.
3. The elected candidate makes a policy choice.

We analyze these choices in reverse order...

# Policy Choice

- The citizen who wins implements her preferred policy, anything else is not credible (no commitment device)

$$\pi_k^* = \operatorname{argmax}_{\pi \in \Pi_k} U_k(\pi, k).$$

- Hence we can associate a utility profile  $(u_{1k}, \dots, u_{Ik})$  to each candidate  $k$  (and to the no candidate situation  $k = 0$ ), with

$$u_{jk} = U_j(\pi_k^*, k)$$

- Let  $\mathcal{C} \subset \mathcal{I}$  be the set of declared candidates.
- Citizens can vote for a candidate in  $\mathcal{C}$  or abstain,  $v_i \in \mathcal{C} \cup \{0\}$ .
- A voting profile  $v = (v_1, \dots, v_I)$ .
- Let  $W(\mathcal{C}, v)$  be the set of winning candidates under  $v$ .
- Then the probability that  $i$  wins the election is

$$P_i(\mathcal{C}, v) = \begin{cases} 1/\#W(\mathcal{C}, v) & \text{if } i \in W(\mathcal{C}, v) \\ 0 & \text{otherwise} \end{cases}$$

# Voting Equilibrium

- In a voting equilibrium, every voter correctly anticipates what policy candidate  $k$  will choose if she wins.
- $v^*$  is a voting equilibrium if (i) and (ii) hold
  - Ⓜ  $v_i^* \in \operatorname{argmax}_{v_i \in C \cup \{0\}} \sum_{k \in C} P_k(C, (v_i, v_{-i}^*)) u_{ik}$
  - Ⓜ  $v_i^*$  is not a weakly dominated voting strategy.
- Note: Ruling out weakly dominated strategies implies sincere voting in two-candidate elections.
- A voting equilibrium exists for any nonempty candidate set. With more than 3 candidates there are multiple equilibria in general.

- Citizen  $i$ 's (pure) entry strategy is  $e_i \in \{0, 1\}$ .
- $e = (e_1, \dots, e_I), \mathcal{C}(e) = \{i | e_i = 1\}$ .
- Suppose  $v(\mathcal{C})$  is the commonly anticipated voting strategy when the set of candidates is  $\mathcal{C}$ .
- Given  $v(\cdot)$  and  $e$ , the expected payoff of  $i$  is

$$U_i(e, v(\cdot)) = \sum_{j \in \mathcal{C}(e)} P_j(\mathcal{C}(e), v(\mathcal{C}(e))) u_{ij} + P_0(\mathcal{C}(e)) u_{i0}$$

where  $P_0(\emptyset) = 1$  and  $P_0(\mathcal{C}) = 0$  otherwise.

We allow for mixed strategy at the entry stage: citizen  $i$  chooses to enter with probability  $\gamma_i$ .

Besley and Coate (1997)

There exists a political equilibrium  $\{\gamma, v(\cdot)\}$

- Typically, there are many equilibria, with one, two or more candidates.

# An Example: Public Good Provision

- $\mathcal{I} = [0, 1]$
- Income  $y_i \sim F(\cdot)$
- $y_m = F^{-1}(1/2) < \bar{y} = \int y dF(y)$
- Preferences:  $u_i(c_i, g) = c_i + H(g)$  with  $H' > 0, H'' < 0$
- Balanced Budget:  $g = \tau \bar{y}$ .
- Hence  $U_i(g) = (1 - g/\bar{y}) y_i + H(g)$



# Voters' ideal policies

- $g_i^* = H'^{-1} \left( \frac{y_i}{y} \right)$
- With Downsian electoral competition the political outcome is  $g_m^*$ .
- What happens with citizen candidates?
- If a citizen runs, she cannot commit to apply anything else than  $g_i^*$ .
- Let  $\delta$  be the entry cost, and  $\hat{g}$  the status quo policy.

# Median Voter

- $U_i(g) = (1 - g/\bar{y}) y_i + H(g)$  satisfies single-peakedness.
- Hence the median voter's preferences determine the outcome of binary electoral contests.

$$U_m(g) > U_m(g') \Rightarrow g \succ^{mv} g'$$

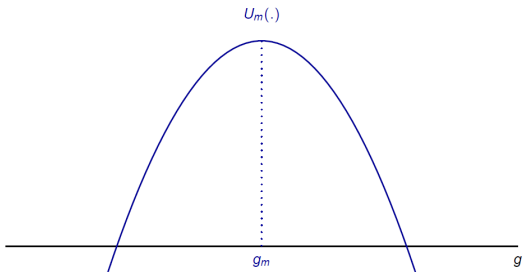
- With more than two competing policies, there is scope for strategic voting...

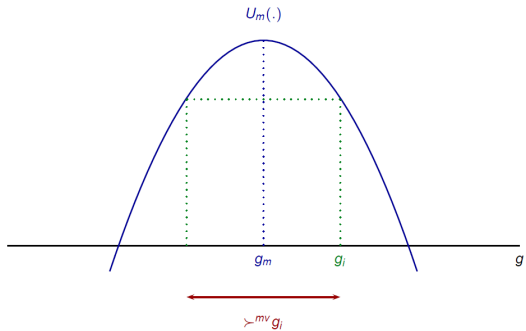
# One-Candidate Equilibria

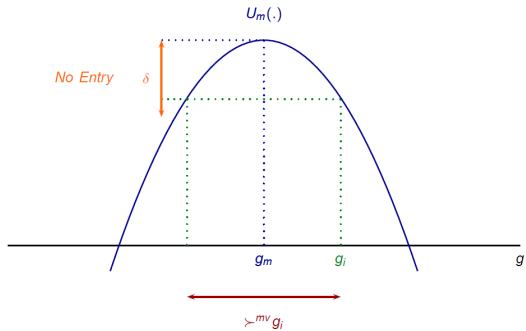
- If the median voter runs, no one can beat him, so no other candidate would enter.
- The median voter wants to run if

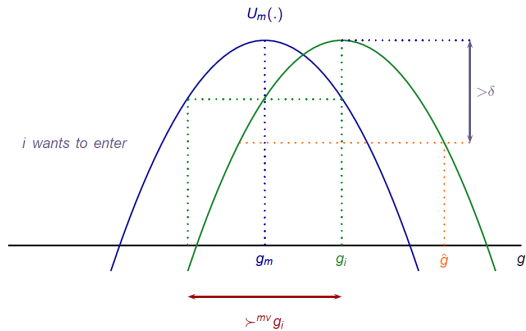
$$U_m(g_m) - \delta > U_m(\hat{g}).$$

- Are there other one-candidate equilibria?









# One-Candidate Equilibria

## Theorem

*There exists an interval  $Y$  of incomes with  $y_m \in Y$  such that for every  $i$  with  $y_i \in Y$  there exists a one-candidate equilibrium in which  $i$  is the candidate, and if  $y_i \notin Y$ , then there is no one-candidate equilibrium such that  $i$  is the candidate.*

- In any one-candidate equilibrium, the political outcome is “close to” the median.



# Two-Candidate Equilibria

- Suppose  $\ell$  and  $r$  are both running.
- Then each of them must stand a chance to win, hence

$$U_m(g_\ell^*) = U_m(g_r^*)$$

- And each of them must prefer to run

$$\frac{1}{2} \{U_\ell(g_\ell^*) + U_\ell(g_r^*)\} - \delta \geq U_\ell(g_r^*) \Rightarrow U_\ell(g_\ell^*) - U_\ell(g_r^*) \geq 2\delta$$

$$\frac{1}{2} \{U_r(g_r^*) + U_r(g_\ell^*)\} - \delta \geq U_r(g_\ell^*) \Rightarrow U_r(g_r^*) - U_r(g_\ell^*) \geq 2\delta$$

- Hence  $g_\ell^*$  and  $g_r^*$  must be sufficiently far. Assume  $g_\ell^* < g_r^*$

# Two-Candidate Equilibria

- No other citizen should be willing to enter...
- Only citizens  $C$  such that  $g_\ell^* < g_C^* < g_r^*$  may have an incentive to enter (why?)
- The pivotal voters are  $i$  and  $j$  such that

$$U_i(g_C^*) = U_i(g_\ell^*),$$

$$U_j(g_C^*) = U_j(g_r^*).$$

- And  $C$  gets  $F(y_j) - F(y_i)$  votes.

# Two-Candidate Equilibria

- C wins if

$$F(y_j) - F(y_i) > \max\{F(y_i), 1 - F(y_j)\} \quad (1)$$

- And C wants to run if

$$U_C(g_C^*) - \delta > \frac{1}{2} \{U_C(g_\ell^*) + U_C(g_r^*)\} \quad (2)$$

Both (1) and (2) tend to be satisfied if  $g_\ell^*$  and  $g_r^*$  are not too close.

# Remarks

- In both one and two-candidate equilibria, the outcome is no longer necessarily the preferred policy of the median voter.
- However, the median voter plays an important role in the characterization of these equilibria.
- In the two-candidate equilibria, a “left” and a “right” party emerge.
- In the one-candidate equilibrium, the status quo plays an important role.
- There are also three-candidate equilibria etc...

- This model does not presume that commitment is possible.
- It is a possible approach towards endogenizing party formation.
- There is a small literature on the formation of political parties (see Morelli, 2004).
- However the multiplicity of equilibria can be a problem.

# To sum up

- We looked at the predictions of the model with sincere voting concerning equilibria with one or two candidates.
- Two candidates with ideologies  $i_A$  and  $i_B$  running against each other is an equilibrium if
  - (i) candidates  $A$  and  $B$  want to run against each other and
  - (ii) there is no third candidate  $C$  who would gain from entering the race.

- In case of sincere voting:
- The first prediction of the model is that  $i_A$  and  $i_B$  will be on opposite sides of the median voter's ideology and the median voter will be indifferent between the two candidates.
- The model also predicts that the ideologies of the two candidates can neither be too far apart nor too close together.
- The basic picture in terms of the positions of the candidates looks very similar to the predictions of the candidate competition model with policy preferences and voter uncertainty.

- If we assume strategic voting, it is no longer the case that the candidates cannot be too far apart.
- With strategic voting even if the two candidates are at the extremes, a centrist candidate would not necessarily be able to enter and win.
- The reason is that in a three way race, centrist voters might continue to view the race as a contest between the two extremist candidates and be reluctant to switch their votes to the centrist candidate for fear of “wasting their votes”.
- This result is notable because it shows that extremism can arise even with a very competitive looking political system.



# Outline of the class

Introduction

**Lecture 2-5: Tools of political economics with applications**

Lecture 6: Comparative Politics

**Part II: Dynamic Political Economy**