Market Design

Pierre Boyer

École Polytechnique

PA SEF

Business Strategies and Finance / Stratégie d'Entreprise et Finance

Fall 2019

Outline of the class

Lecture 1: Introduction

Lecture 2: Auction theory and design

Lecture 3: Common-value and Multi-unit auctions

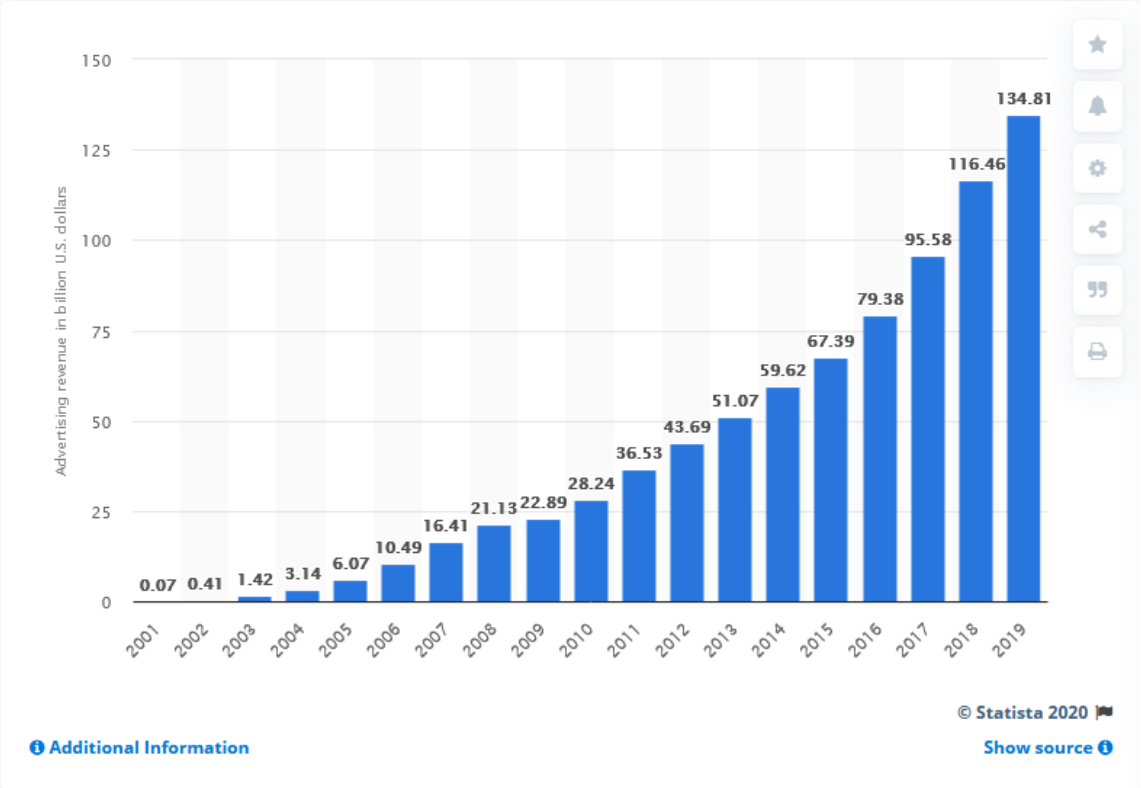
Lecture 4: Multi-item auctions and matching, sponsored-search auctions, spectrum auctions, package auctions

Sponsored Search Auctions

- Search advertising is a huge auction market.
- Why would you use auctions in this setting?
  - Difficult to set so many prices (tens of millions of keywords).
  - Demand and especially supply might be changing.
  - Retain some price-setting ability via auction design.

Advertising revenue of Google from 2001 to 2019

(in billion U.S. dollars)



Keyword Auctions

- Advertisers submit bids for keywords
  - E.g. offer a dollar payment per click.
- Separate auction for every query
  - Positions awarded in order of bid (more on this later).
  - Google uses "generalized second price" auction format (GSP).
  - Advertisers pay bid of the advertiser in the position below.
- Some important features
  - Value is created by getting a good match of ad to searcher.
  - Multiple positions, but advertisers submit only a single bid.

A Brief History

- Early 1990s: Websites sell advertising space on a "per-eyeball" basis, with contracts negotiated by salespeople; similar to print or television.
- Mid 1990s: Overture Services, Inc. (formerly GoTo.com) allows advertisers to bid for keywords, offering to pay per click. Yahoo! and others adopt this approach, charging advertisers their bids.
- 2000s: Google and Overture modify keyword auction to have advertisers pay minimum amount necessary to maintain their position (GSP).
- Late 2000s: Auction design becomes more sophisticated; auctions used to allocate advertising on many webpages (e.g. Facebook), not just search.

A Simplified Model

- Positions $k = 1, \cdots, K$ Bidders $n = 1, \cdots, N$
- Positions are ordered according to the numbers of clicks-per-day they receive: $x_1 > \cdots > x_K$
- Bidders are ordered according to their per-click value:

$$v_1 > \cdots > v_n$$

- Bidder n value for position k is $v_n x_k$.
- Bidder n profit from position k with a per-click-price of p_k is

$$(v_n - p_k)x_k$$

- The efficient or surplus maximizing assignment is assortative: (1,1); (2,2)...

Example

- Two positions receive 200 and 100 clicks per day (cpd).
- 3 bidders with per click values (pcv) \$10, \$4, \$2. One position per bidder.

	Top (200 cpd)	2nd (100 cpd)
Bidder 1	2000	1000
Bidder 2	800	400
Bidder 3	400	200

- Efficient allocation creates value \$2400
- Can we find per-click prices for the two positions that clear this market?

Market Clearing Prices

- Solve for lowest market clearing prices

	Top (200 cpd)	2nd (100 cpd)
Bidder 1	2000	1000
Bidder 2	800	400
Bidder 3	400	200

- Lowest market clearing prices: $P_1 = 600$ and $P_2 = 200$
  - Bidder 1 prefers top position.
  - Bidder 2 prefers 2nd position.
  - Bidder 3 prefers to stay out.
- Corresponding per-click prices: $p_1 = 3$ and $p_2 = 2$.
- These prices achieve the efficient allocation and generate revenue of \$800.

Market Clearing Prices

- Solve for all market clearing prices

	Top (200 cpd)	2nd (100 cpd)
Bidder 1 (10 pcv)	2000	1000
Bidder 2 (4 pcv)	800	400
Bidder 3 (2 pcv)	400	200

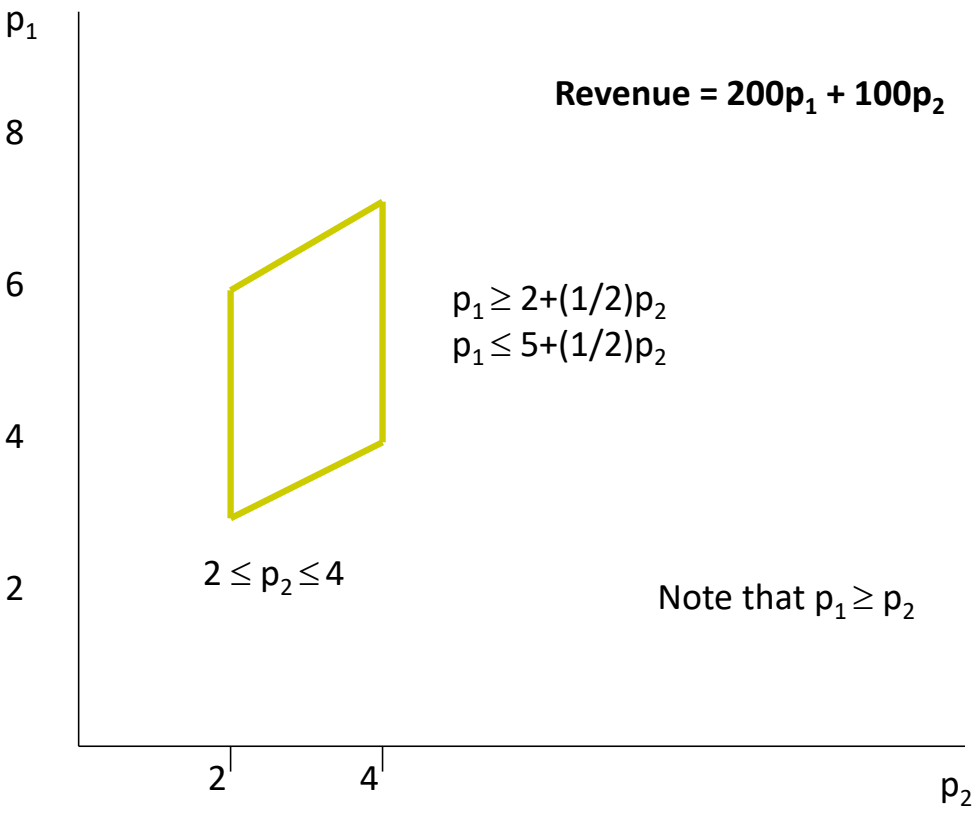
- Bidder 3 prefers nothing: $p_1 \geq 2 \quad \text{and} \quad p_2 \geq 2$
- Bidder 2 prefers 2nd: $p_2 \leq 4$ and

$$200(4 - p_1) \leq 100(4 - p_2) \quad \Rightarrow \quad p_1 \geq 2 + \frac{1}{2}p_2$$

- Bidder 1 prefers top: $p_1 \leq 10$ (redundant) and

$$200(10 - p_1) > 100(10 - p_2) \quad \Rightarrow \quad p_1 < 5 + \frac{1}{2}p_2$$

Market Clearing Prices



1

Price premium for more clicks

- At market clearing prices, bidder k prefers position k to $k-1$:

$$x_k(v_k - p_k) \geq x_{k-1}(v_k - p_{k-1}) \quad \Rightarrow \quad p_{k-1} \geq \underbrace{v_k}_{\geq p_k} \underbrace{\left(1 - \frac{x_k}{x_{k-1}}\right)}_{>0} + \frac{x_k}{x_{k-1}} p_k$$

$$\Rightarrow \quad p_{k-1} \geq p_k$$

- Positions with more clicks get higher per click prices.

Finding Market Clearing Prices

Find lowest prices:

- Set $p_K = v_{K+1}$ so bidder $K+1$ is indifferent between K and nothing.
- Set p_{k-1} so that bidder k is indifferent between k and $k-1$:

$$x_{k-1}(v_k - p_{k-1}) = x_k(v_k - p_k)$$

Find highest prices:

- Set $p_K = v_K$ so bidder K is indifferent between K and nothing.
- Set p_{k-1} so that bidder $k-1$ is indifferent between k and $k-1$:

$$x_{k-1}(v_{k-1} - p_{k-1}) = x_k(v_{k-1} - p_k)$$

Sponsored Search Auction

- Can we design an auction to find market clearing prices?
- Ideally want to use the structure of the problem to design a simple auction. We will consider several options.

Pay-as-Bid Auction

Overture "Pay-as-Bid" Auction:

- Each bidder submits a single bid (in $ per click)
- Top bid gets position 1, second bid position 2, etc.
- Bidders pay their bid for each click they get.

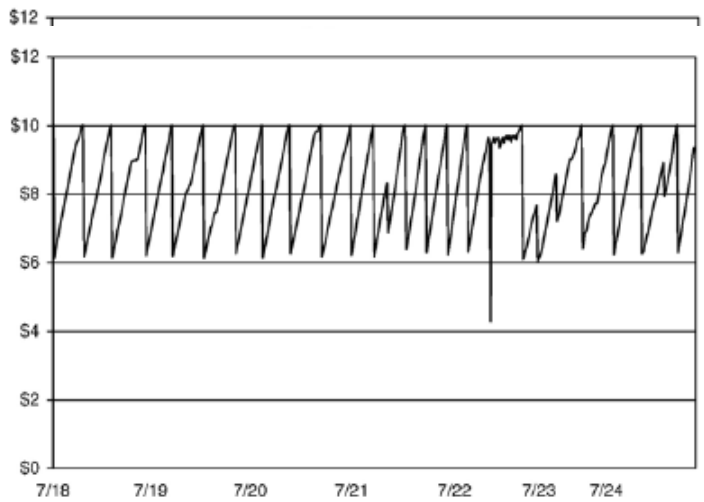
Pay-as-Bid Example

	Top (200 cpd)	2nd (100 cpd)
Bidder 1 (10 pcv)	2000	1000
Bidder 2 (4 pcv)	800	400
Bidder 3 (2 pcv)	400	200

- Bidder 3 will offer up to \$2 per click.
- Bidder 2 wants to bid \$2.01 to get second slot.
- Bidder 1 wants to bid \$2.02 to get first slot.
- But then bidder 2 wants to bid \$2.03 and get first slot...

Pay as bid auction is unstable

Overture Bid Pattern



(b) 1 week

Edelman and Ostrovsky (2006): "sawtooth" pattern caused by auto-bidding programs.

Google GSP Auction

Generalized Second Price Auction:

- Bidders submit bids (in $ per click)
- Top bid gets slot 1, second bid gets slot 2, etc.
- Each bidder pays the bid of the bidder below him.
- Seems intuitively like a more stable auction.
- Do the bidders want to bid truthfully?

Truthful Bidding

It is not a dominant strategy to bid truthfully

Example:

- Two positions: $x_1 = 200$ and $x_2 = 100$.
- Consider a bidder with value 10
- Suppose competing bids are 8 and 4
  - Bid 10 $\Rightarrow$ $200(10-8) = 400$
  - Bid 7 $\Rightarrow$ $100(10-4) = 600$

GSP Auction: Example

	Top (200 cpd)	2nd (100 cpd)
Bidder 1 (10 pcv)	2000	1000
Bidder 2 (4 pcv)	800	400
Bidder 3 (2 pcv)	400	200

- In this example, truthful bidding (10,4,2) is a Nash equilibrium (not true in general):
  - Bidder 3 would have to pay 4 to get slot 2 $\Rightarrow \quad -200 < 0$
  - Bidder 2 gets slot 2 and profit $200 > 0$, if bid 10 to get slot 1, profit $-1200 < 200$
  - Bidder 1 has profit 1200, if bid 4 to get slot 2

    $\Rightarrow \quad (10-2)100 < 1200.$

GSP Auction: Example

	Top (200 cpd)	2nd (100 cpd)
Bidder 1 (10 pcv)	2000	1000
Bidder 2 (4 pcv)	800	400
Bidder 3 (2 pcv)	400	200

- This is not the only NE however: (10,7,4) is also a NE
  - Bidder 3 cannot do better than 0 by changing bid.
  - Bidder 2 is making 0 profit, but would get -600 if changed his bid to 10.
  - Bidder 1 makes 600 profit, and would get $(10-4)100 = 600$ if changed bid to 7 to get slot 2.
- This equilibrium achieves the highest market clearing prices, i.e. highest revenue.

GSP Equilibria

Assumption: $N > K$.

Theorem. For every set of market clearing prices $p_1, \cdots, p_K$, there exists a NE of the GSP auction that achieves these prices.

Proof.

- Bidder 1 bids his value, bidder 2 bids p_1, . . ., bidder $K+1$ bids p_K.
- Since the prices are market clearing, bidder k prefers buying position k than any other position m at these prices.

Vickrey Auction

- Bidders submit bids ($ per-click)
- Seller finds assignment that maximizes total value assuming truthful bids
  - Puts highest bidder in top position, next in 2nd slot, etc.
- Charges each winner the total value their bid displaces.
  - For bidder n, each bidder below n is displaced by one position, so must add up the value of all these "lost" clicks.
- Note that Facebook uses a Vickrey auction.
- Dominant strategy to bid truthfully

Vickrey Auction Prices

- Bids: $b_1 > b_2 > \cdots > b_N$.
- Bidder k displaces bidders $k+1, \cdots, K+1$.
- Without k, each bidder $k+1 < l < K+1$ would get $x_{l-1} b_l$ instead of $x_l b_l$.
- Hence k should pay :

$$x_k p_k^V = \sum_{l=k+1}^{K+1} b_l \left(x_{l-1} - x_l\right) \qquad \text{where } x_{K+1} = 0$$

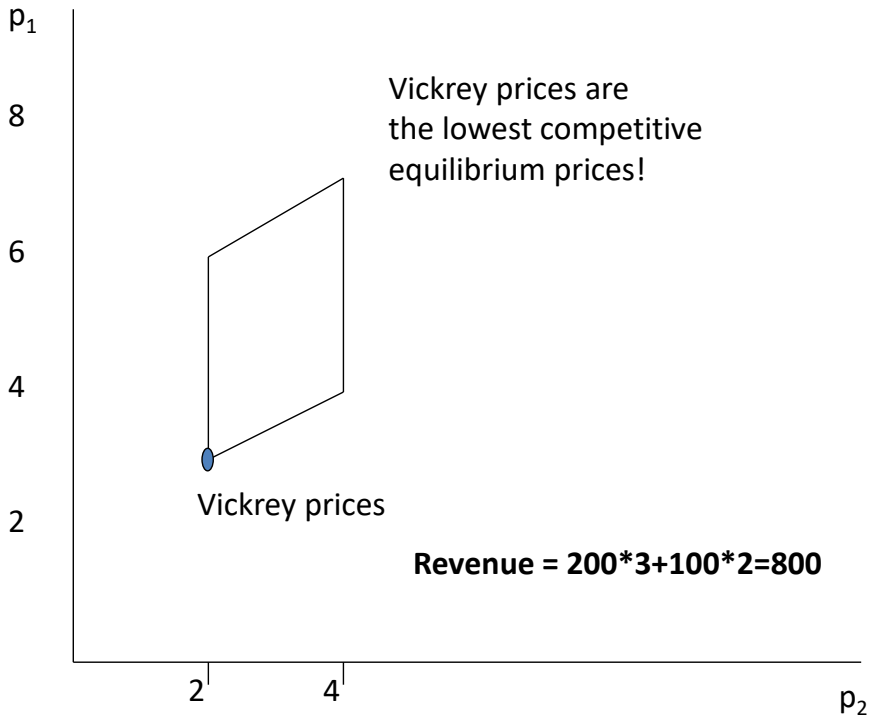
Theorem. Vickrey prices = lowest market clearing prices.

Vickrey Auction: Example

	Top (200 cpd)	2nd (100 cpd)
Bidder 1 (10 pcv)	2000	1000
Bidder 2 (4 pcv)	800	400
Bidder 3 (2 pcv)	400	200

- **Truthful bids:** 10,4,2
- Bidder 2 pays 200 for displacing bidder 3 (=$2 per click).
- Bidder 1 pays 200 for displacing bidder 3 + 800-400 for displacing bidder 2 to slot 2 =600 ($3 per click)

Vickrey Auction: Example



1

Summary of Results

GSP Auction:

- Truthful bidding not a dominant strategy or an equilibrium in general.
- All market clearing prices are equilibrium prices (equivalent if we rule out strange equilibria).
- Edelman, Ostrovsky and Schwartz (2007, AER) argue that dynamics converge to lowest market clearing prices

Vickrey Auction:

- Truthful bidding is a dominant strategy.
- Vickrey prices are the lowest market clearing prices.
- Price computation is harder to understand for bidders.

Keyword Auction Design

- Platforms do retain some control over prices
  - Restricting the number of slots can increase prices.
  - Setting a reserve price can increase prices
- Platforms can also "quality-adjust" bids
  - In practice, ads that are more "clickable" get promoted.
  - Bids can be ranked according to score = bid $\times$ quality.
  - This gives an advantage to high-quality advertisements.

Quality Scoring

- Suppose that instead of any bidder getting x_k clicks in position k, bidder n can expect to get $a_n x_k$ clicks.
- If a bidder has a high a_n, its ad is "clickable".
- In practice, Google and Bing run giant regressions to try to estimate the "clickability" of different ads.
- Then bids in the auction can be ranked by $a_n b_n$, which means that clickable ads get prioritized in the rankings.
- This can have advantages and disadvantages
  - Puts weight on what users want and rewards higher quality ads.
  - Sometimes can reduce revenue if one ad gets lots of clicks.
- Also need to run tests to learn about "clickability".

Sponsored vs Organic Results

- Google and Bing show "organic" search results and "sponsored" in different places.
  - The assignment of positions on the page is different
  - Organic search results: use algorithm to assess "relevance"
  - Sponsored search results: use bids to assess "value"
- To some extent there is competition
  - If a site gets a good organic position, should it pay for another?
  - Search engines have to think about maximizing user experience but also about capturing revenue from advertisers.

Summary: Sponsored Search

- Search auctions create a real-time market in which advertising opportunities are allocated to bidders.
- Auction theory suggests why the "second-price" rules used in practice might be reasonably efficient.
  - GSP does not induce "truthful" bidding but it has efficient Nash equilibria with competitive prices.
  - Vickrey auction does induce truthful bidding, but prices depend on a more complicated formula

Outline of the class

Lecture 1: Introduction

Lecture 2: Auction theory and design

Lecture 3: Common-value and Multi-unit auctions

Lecture 4: Multi-item auctions and matching, sponsored-search auctions, spectrum auctions, package auctions