

Market Design

Pierre Boyer

École Polytechnique

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Outline of the class

Lecture 1: Introduction

Lecture 2: Auction theory and design

Lecture 3: Common-value and Multi-unit auctions

Lecture 4: Multi-item auctions and matching, sponsored-search auctions, spectrum auctions, package auctions

Common-value auctions

Example: Coin Auction

- Item for sale: the coins in a jar.
- Ascending Auction.

Common-value auctions

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- Ascending Auction.
- What if one of you gets to look in the jar?
- Does this change your bid? Why?

Common-value auctions

Example: Coin Auction

- Item for sale: the coins in a jar.
- Ascending Auction.
- What if one of you gets to look in the jar?
- Does this change your bid? Why?
- What if all of you get to look in the jar?

The Winner's Curse

- Winning the jar means that everyone else in the class was more pessimistic about its contents.
- Winning is “bad news”:
 - ▶ If you had an initial estimate of \$10, knowing that everyone else had a lower estimate should cause you to revise your estimate downward.
- Equilibrium bidding should account for this.

Common Value Auctions: Second-price auction

- The “imperfect estimate” model or interdependent value model:
 - ▶ Two bidders with common value v drawn at random from a distribution (*pure* common value).
 - ▶ Each bidder observes a signal s_i correlated with v
 - ▶ Signals are iid and provide imperfect information about v
 - ▶ Assumption: $E(v|s_1, s_2) = E(v|s_2, s_1)$ (symmetry) is increasing in s_1 and s_2 .
- Example: $s_i = v + \varepsilon_i$ where $\varepsilon_i \sim \mathcal{N}(0, \sigma)$.

The Winner's curse (formal example)

- Prior to the auction, the only information available to the bidder i is s_i .
 - Based on this information, estimate of value v is $E(v|s_1)$.
 - If all bidders symmetric and follow the same strategy, winning the auction reveals other bidder signal lower than s_i .
 - Now estimate of the value if winner: $E(v|s_1, s_2 < s_1) < E(v|s_1)$.
- ⇒ Winning is bad news: winner's curse
- This is going to be internalized in the equilibrium bidding behavior.

Common Value Auctions: Second-price auction

- Bidder i submits bid $b_i(s_1, s_2)$: now valuation depends on each other.
- Remark: in interdependent value models, English (ascending) auction and second-price auction equivalent only if two bidders
- We look for a symmetric equilibrium $b(s)$ with increasing and differentiable s .

How should the bidders account for the winner's curse in equilibrium?

Equilibrium Bidding

Expected profit of a bidder i with signal s_i :

$$U_i(s_i) = \{E(v \mid s_i, b_j < b_i) - b_j\} \Pr(b_j < b_i \mid s_i)$$

From that, we could work to find the symmetric BNE of this game...
but there is a more intuitive way to think about it.

Claim: In the equilibrium of the second-price auction, the bidders use the strategy:

$$b_i^*(s_i, s_j) = E(v \mid s_i, s_j)$$

Idea of the proof:

- If b_i^* is an equilibrium strategy, then i prefers $b_i^*(s_i, s_j)$ to $b(s_i + \delta, s_j)$ for any small δ .
- Such a small change in the bid will matter if and only if j bids exactly $E(v \mid s_i, s_j)$, that is if and only if j has received the same signal as i .
- In equilibrium, i must not want to change her bid even in that case.
- For that to be true, i 's bid must be equal to her estimate of v in that precise case, that is $E(v \mid s_i, s_j = s_i)$.

- N bidders with symmetric signals s_1, \dots, s_N
- Define

$$\tilde{s}_1 = \max_{j=2, \dots, n} s_j$$

and

$$v(x, y) = E(v | s_1 = x, \tilde{s}_1 = y)$$

Theorem. The symmetric equilibrium bidding function is given by

$$\beta(s) = v(s, s).$$

Key idea: Bidders must account that winning (or losing conveys) information about the information of other bidders.

Winner's Curse or Loser's Curse?

Does accounting for the information of others mean that you must bid higher or lower than your baseline estimate?

- **First Case:** 1 item, 10 bidders, second-price auction:
 - ▶ Winning means other signals were all lower.
 - ⇒ There is a winner's curse.
- **Second Case:** 8 items, 10 bidders, ninth price auction:
 - ▶ Losing means other signals were all higher.
 - ⇒ There is a loser's curse.

Revenue Equivalence?

If bidders have “correlated” estimates of a common value item, an open ascending auction leads to a higher revenue than a sealed-bid second-price auction, which leads to a higher revenue than a first-price sealed bid auction. (Milgrom-Weber)

Intuition:

- In either case, the bidder with the highest estimate will win.
- In sealed bid FPA, the payment will be independent of the estimate of the second highest bidder.
- In the ascending auction or SPA, the payment will depend on the second highest estimate.
- The bidder estimates are correlated.
- So the open auction sets the price for the winner high when the other bidder has good news and low when the other bidder has bad news.
- This reduces the winner's profit and increases the seller's revenue.

Linkage Principle

There is a broader principle here: If the seller can give bidders access to better information (by inspecting the item for example), then the revenue is increased on average by making the information publicly available.

Intuition:

- The public information can either increase or decrease everyone's bid (depending on whether the news is good or bad).
- The public information will tend to be good news exactly when the high bidder has a high value.
- So releasing information “squeezes” the high bidder in the right cases.

Providing Information to Bidders

- According to the linkage principle, the seller should commit ex ante to publicly reveal any information about the value of the item.
- But in some cases, giving bidders the opportunity to acquire information can create informational asymmetries

What kinds of auctions have a “common value” flavor to them?

- Timber auctions: quality of the timber being sold.
- Oil lease auctions: quantity of oil. Bidders do independent studies so each has valuable information.

Outer Continental Shelf Auctions

- The US government auctions the right to drill for oil on the outer continental shelf.
- Value of oil is similar to the different bidders, but no one knows how much oil there is, or if there is none.
- Prior to the auction, the bidders do seismic studies.
- Two kinds of sale
 - ▶ “Wildcat sale” - new territory being sold
 - ▶ “Drainage sale” - territory adjacent to existing tract.

Wildcat vs Drainage

TABLE 1—SELECTED STATISTICS ON WILDCAT
AND DRAINAGE TRACTS^a

	Wildcat	Drainage
Number of Tracts	1056	144
Number of Tracts Drilled	748	124
Number of Productive Tracts	385	86
Average Winning Bid	2.67 (0.18)	5.76 (1.07)
Average Net Profits	1.22 (0.50)	4.63 (1.59)
Average Tract Value	5.27 (0.64)	13.51 (2.84)
Average Number of Bidders	3.46	2.73

^a*Source:* Kenneth Hendricks, Robert Porter, and Bryan Boudreau (1987). Dollar figures are in millions of \$1972. The numbers in parentheses are standard deviations of the sample means.

TABLE 3—SAMPLE STATISTICS ON TRACTS WON BY EACH TYPE OF FIRM^a

	Wins by Neighbor Firms		Wins by Non-Neighbor Firms		
	A	Total	B	C	Total
No. of Tracts	35	59	19	36	55
No. of Tracts Drilled	23	47	18	33	51
No. of Productive Tracts	16	36	12	19	31
Average Winning Bid	3.28	6.04	2.15	6.30	4.87
	(0.56)	(2.00)	(0.67)	(1.31)	(0.92)
Average Gross Profits	10.05	12.75	-0.54	7.08	4.45
	(3.91)	(3.21)	(0.47)	(2.95)	(1.99)
Average Net Profits	6.76	6.71	-2.69	0.78	-0.42
	(3.02)	(2.69)	(0.86)	(2.64)	(1.76)

^a Dollar figures are in millions of \$1972. The numbers in parentheses are the standard deviations of the sample means. Column A refers to tracts which received no non-neighbor firm bid, column B refers to tracts which received no neighbor bid, and column C to those in which a neighbor firm bid, but a non-neighbor firm won the tract.

Explaining the results

Comparing wildcat and drainage sales:

- Wildcat sales yield low profits \Rightarrow competition.
- Drainage sales are profitable, but only for “insiders” \Rightarrow insiders have an advantage

Initial Public Offerings

- In an IPO, all buyers essentially have the same value v , the stock price once trading opens.
- You should want to buy in the IPO if you think the IPO price p is less than v .
- Do IPOs sell at a discount? Not necessarily. If everyone know v , competition and “no arbitrage” should drive price up to v .

Average first-day returns relative to file price range:

	Below	Within	Above
1980-1989	0%	6%	20%
1990-1998	4%	11%	31%
1999-2000	8%	26%	121%
2001-2015	3%	11%	36%
1980-2015	3%	11%	50%

Source: Jay Ritter, University of Florida

- One explanation is the winner's curse.
 - ▶ If some potential buyers are informed, then if you obtain shares in the IPO, it may mean the informed buyers stayed away.
 - ▶ Therefore, regular buyers may need to be cautious, and as a result, IPOs sell at a discount.

- There are alternative explanations
 - ▶ Informed banks set IPO prices low to cater to clients.
 - ▶ Informed banks deliberately choose a low price to have control over who gets allocated shares, and avoid any risk of under-sell.

- Perhaps well-designed IPO auction could aggregate information, but IPOs generally don't use auctions...

Summary

- Many auctions have a “common value” flavor.
- In common value settings:
 - ▶ The event of winning reveals information about opponent estimates, and bidders must account for this.
 - ▶ Bidders without accurate information must be cautious when bidding against bidders with very good information.
 - ▶ If there are many bidders and dispersed information, the auction price can be a useful indicator of the item’s value.
 - ▶ The management of information is very important.

Multi-Unit Auctions

- Many auctions involve the sale of multiple similar or identical units.
 - ▶ Cases of wine, carbon permits, shares of a company, treasury bills, megawatts of electricity, etc.
- Different approaches to multi-unit auctions
 - ▶ Sequential sales vs simultaneous sale
 - ▶ Clock auction vs sealed bidding
 - ▶ Uniform price vs discriminatory price vs Vickrey pricing
- We look at the design of multi-unit auctions, with practical examples.

Sequential Auctions

- Auction houses often sell identical goods sequentially (e.g. cases of wine).
- What happens at sequential auctions?
- Should you bid your value in the first auction?
- Are early prices higher or lower than later prices?

Table 1
Typical Price Sequences for Identical Wines
Sold in a Single Auction
(in £ sterling per dozen bottles)

<i>Wine Type</i>	<i>Chateau Palmer 1961</i>		<i>Croft (Port) 1927</i>		<i>Chateau Margaux 1952</i>		<i>Quinta de Noral (Port) 1934</i>	
	<i>Lot Size</i>	<i>Price</i>	<i>Lot Size</i>	<i>Price</i>	<i>Lot Size</i>	<i>Price</i>	<i>Lot Size</i>	<i>Price</i>
Lot 1	12 bts	920	12 bts	800	12 bts	480	10 bts	400
Lot 2	12 bts	800	12 bts	800	12 bts	480	12 bts	500
Lot 3	12 bts	700	12 bts	750	12 bts	480	12 bts	500
Lot 4			12 bts	650	24 bts	480	12 bts	480
Lot 5			12 bts	650	24 bts	480	12 bts	480
Lot 6			12 bts	650	20 bts	480		
Lot 7			12 bts	650				

Source: Author's tabulation of the results of Sotheby's Auction of Finest and Rarest wines, December 11, 1985.

Source: Ashenfelter (1989, Journal of Economic Perspectives)

Why do prices decline?

- Ginsburgh (1998, JPE) provides an explanation based on Sotheby's wine auctions: many bidders are absentee and give instructions "bid up to X for one case of Y"

Bidder	Max Bid
A	140
B	120
C	107
D	95

- With 3 lots, the price declines: $120 \rightarrow 107 \rightarrow 95$

Equilibrium bidding

- If bidders are more sophisticated, declining prices cannot be an equilibrium:
 - ▶ Sophisticated bidders would optimally respond by sitting back in early auctions and bidding more aggressively in later ones
- However, the declining price anomaly appears in many auctions (art, cattle, wool, etc.) and remains somewhat of a puzzle.

Simultaneous Auctions

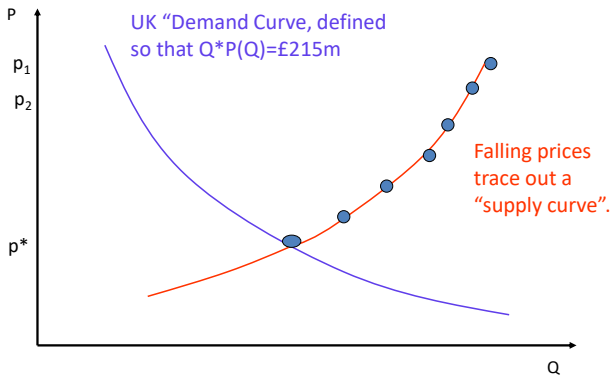
- Consider selling k identical items through a simultaneous auction.
- Multiple possible designs:
 - ▶ Uniform price: all items are sold at the same price (clock and sealed-bid auctions).
 - ▶ Discriminatory price (pay-your-bid or Vickrey).
- One important issue is whether bidders are interested in only one unit or more.

Uniform price auctions

- Sellers often want to run auctions in which all winners pay the same uniform price.
- Perceived as “fair” and achieves price discovery.
- **Uniform price formats:**
 - ▶ **Clock auction:** seller announces a sequence of prices and bidders name quantities until a market clearing price is found, then auction ends.
 - ▶ **Sealed bidding:** participants bid a price-quantity schedule and bids are used to determine the uniform market clearing price.

A Clock Auction: British CO₂ Auction

- In 2002, the British government decided to spend £215 million paying firms to reduce CO₂ emissions.
 - ▶ But what price to pay per unit? And which firms to reward?
 - ▶ Solution: run an auction to find the “market price”.
- **Greenhouse Gas Emissions Trading Scheme Auction:**
 - ▶ Per unit price starts high and decreases each round.
 - ▶ Each round, bidders state tons of CO₂ they will abate at this price.
 - ▶ Cost to UK: (Tons of Abatement) \times (Price).
 - ▶ Auction ended when total cost equaled the budget.
- Result: 34 firms paid to reduce emissions by a total of 4 million metric tons of CO₂.



1

Uniform-price sealed bid auction:

- Auctioneer posts its demand curve
- Bidders submit “supply curves” - i.e. how much they will supply at each price.
- Individual supply curves are aggregated to form an aggregate supply curve.
- Price is set so that supply = demand

Sealed Bid vs Clock Auctions: Does it matter?

- Depends on the information released to bidders
- Suppose bidders in the clock auction observe only the prices and that prices decline in a fixed sequence.
- Bidders are being asked to reveal their supply curves from the top down, with no new information each round other than that the current price is relevant.
- So the auction is strategically equivalent to sealed bidding in which supply curves are written down in advance.
- Of course, if more information is released during a clock auction, bidders can adjust their bidding in response to competition - why might this happen?

Google's IPO Auction

- Uniform price sealed bid.
- Bidders submit demands for shares at different prices.
- Bankers construct market demand curve and intersect with supply ($\sim 20\text{M}$ shares).

- Final price: \$85
- In the first day of trading, the price went up to \$100: auction failure?
- In fact, informed Bankers decided to lower the price from true auction price.

Incentives in Uniform Price Auctions

- N bidders, K identical items for sale.
- Uniform price auction. Either:
 - ▶ Simultaneous: final price is $K + 1^{\text{st}}$ price.
 - ▶ Clock: stops when demand is equal to K .
- **Theorem:** For a bidder with single-unit demand, it is a weakly dominant strategy to bid truthfully.
- What about other bidders?

Demand Reduction

- $N = 4$ bidders, $K = 3$ units.
- Values:

Bidder	1 st unit	2 nd unit	3 rd unit
A	110	0	0
B	105	0	0
C	100	100	0
D	90	0	0

- Truthful bidding: A: 1@110; B: 1@105; C: 2@100; D: 1@90.
- Outcome: A, B and C win 1 unit each, price is 100.

Demand Reduction

- $N = 4$ bidders, $K = 3$ units.
- Values:

Bidder	1 st unit	2 nd unit	3 rd unit
A	110	0	0
B	105	0	0
C	100	100	0
D	90	0	0

- Demand Reduction: A: 1@110; B: 1@105; C: 1@100; D: 1@90.
- Outcome: A, B and C win 1 unit each, price is 90.

Uniform Price Auctions: Summary

- Uniform price auctions have desirable properties
 - ▶ Fairness: identical goods sell for identical prices
 - ▶ Simplicity: auction price equates demand and supply.
- Demand reduction is a primary concern
 - ▶ If bidders want more than one unit, they have an incentive to bid less than their true demand in order to reduce price.
 - ▶ Demand reduction can also interfere with efficiency: a standard problem when firms exercise market power.
 - ▶ If supply is inelastic, it can also lead to very low prices - a possibility in both clock and sealed bid auctions.

Discriminatory Auctions: Pay-as-Bid

- Bidders submit bids (demand curves)
- Seller finds price where supply=demand
- All bids above clearing price are satisfied, but winners pay their bid rather than the clearing price.
- How should bidders adapt their bidding?

Pay-as-Bid: Example 1

Bidder	1 st unit	2 nd unit	3 rd unit
A	110	0	0
B	105	0	0
C	100	0	0
D	90	0	0

- Truthful bidding: A: 1@110; B: 1@105; C: 1@100; D: 1@90
- Outcome: A,B,C win and pay 110, 105, 100
- Is this an equilibrium?

Pay-as-Bid: Example 1

Bidder	1 st unit	2 nd unit	3 rd unit
A	110	0	0
B	105	0	0
C	100	0	0
D	90	0	0

- Truthful bidding: A: 1@110; B: 1@105; C: 1@100; D: 1@90
- Outcome: A,B,C win and pay 110, 105, 100
- Is this an equilibrium?
- An equilibrium is A,B,C 1@90; D:1@89.

Pay-as-Bid: Example 2

Bidder	1 st unit	2 nd unit	3 rd unit
A	110	0	0
B	105	0	0
C	100	100	0
D	90	0	0

- Truthful bidding: A: 1@110; B: 1@105; C: 2@100; D: 1@90
- Outcome: A,B,C win 1 unit and pay 110, 105, 100
- Is this an equilibrium?

Pay-as-Bid: Example 2

Bidder	1 st unit	2 nd unit	3 rd unit
A	110	0	0
B	105	0	0
C	100	100	0
D	90	0	0

- Truthful bidding: A: 1@110; B: 1@105; C: 2@100; D: 1@90
- Outcome: A,B,C win 1 unit and pay 110, 105, 100
- Is this an equilibrium?
- What about ? A,B: 1@91; C: 2@90;D: 1@89.

Pay-as-Bid: Example 2

Bidder	1 st unit	2 nd unit	3 rd unit
A	110	0	0
B	105	0	0
C	100	100	0
D	90	0	0

- Truthful bidding: A: 1@110; B: 1@105; C: 2@100; D: 1@90
- Outcome: A,B,C win 1 unit and pay 110, 105, 100
- Is this an equilibrium?
- What about ? A,B: 1@91; C: 2@90;D: 1@89.
- No: C deviates to 2@92
- Equilibrium involves mixed strategies.

Discriminatory Auctions Summary

- Naive view of discriminatory auctions: opportunity to extract more money from “high-value” bidders.
- But bidders compensate with more “demand reduction” – bid “flatter” demand curves that anticipate clearing price.
- Comparison with uniform price auction is tricky
 - ▶ Both auctions can be inefficient and encourage demand reduction: no clear efficiency or price ranking.
 - ▶ Sometimes hear that uniform price is better for small bidders: easier to participate and get the “market price”.

Treasury Auctions

- Long-standing debate on uniform vs discriminatory.
- US used discriminatory until 1992, then switched.
- Studies of change do not find big differences
 - ▶ In both cases, auction prices are quite similar to prices before the auction in the “when issued” market.
 - ▶ Some evidence that smaller bidders increased their market share after switch to uniform price auction.
- Explanation? US market is very large and liquid. Maybe rules matter more when market is thinner.

Efficient Auction?

Bidder	1 st unit	2 nd unit	3 rd unit
A	110	0	0
B	105	0	0
C	100	100	0
D	90	0	0

Is there a pricing rule that would lead to an efficient outcome and such that it would be a dominant strategy for each bidder to bid truthfully?

Vickrey Auction

Bidder	1 st unit	2 nd unit	3 rd unit
A	110	0	0
B	105	0	0
C	100	100	0
D	90	0	0

- Interpret bids as values and choose the allocation to maximize surplus.
- Set the price paid by each winning bidder to the value of losing bids that the bidder displaces.
- These prices make truthful bidding a dominant strategy, but they are not uniform.

Vickrey Auction

Bidder	1 st unit	2 nd unit	3 rd unit
A	110	0	0
B	105	0	0
C	100	100	0
D	90	0	0

- Truthful bidding: A: 1@110; B: 1@105; C: 2@100; D: 1@90
- Outcome: A,B,C win 1 unit and pay 100, 100, 90
 - ▶ A and B displace C for a second unit \rightarrow pay 100.
 - ▶ C displaces D \rightarrow pay 90.

Vickrey Auction

Bidder	1 st unit	2 nd unit	3 rd unit
A	110	0	0
B	105	0	0
C	107	107	0
D	90	0	0

- Truthful bidding: A: 1@110; B: 1@105; C: 2@107; D: 1@90
- Outcome: A,C win 1 and 2 units and pay 105, 105, 90
 - ▶ A displaces B for a unit → pay 105.
 - ▶ C displaces B and D for one unit each → pay 105 for 1st unit and 90 for 2nd.

Summary

- Multiple units can be sold sequentially or simultaneously.
 - Sequential auctions can be simple - one unit at a time.
 - Simultaneous auctions can be designed to yield a uniform price.
- Uniform price auctions can lead to concerns about the exercise of market power
 - Demand reduction when bidders want multiple units
 - Possibility of low price “collusive seeming” equilibria.
- Discriminatory price auctions are an alternative, and are sometimes viewed as a way to extract value from high value bidders, but revenue implications generally unclear.
- Vickrey auction can eliminate demand reduction and restore efficiency, but the uniform price property is lost.

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Lecture 3: Common-value and Multi-unit auctions

Lecture 4: Multi-item auctions and matching, sponsored-search auctions, spectrum auctions, package auctions