Market Design

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Outline of the class

Lecture 1: Introduction

Lecture 2: Auction theory and design

Lecture 3: Common-value and Multi-unit auctions

Lecture 4: Multi-item auctions and matching, sponsored-search auctions, spectrum auctions, package auctions

Auction theory: The classics

Five classical auction designs:

1. Open Ascending Price Auction (English auction).
2. Second-Price Sealed-Bid Auction and Vickrey auction.
3. First-Price Sealed-Bid Auction.
4. Descending Price Auction (Dutch auction).
5. All-Pay Auction.

References

- Milgrom, P. R. (2004). Putting auction theory to work. Cambridge University Press.
- Krishna, V. (2009). Auction theory. Academic press.
- Klemperer, P. (2004). Auctions: Theory and Practice. Princeton University Press.

Play: Second Price Sealed-Bid Auction

- Bidders submit sealed bids.
- The seller opens the bids.
  - The bidder with the highest bid wins the auction.
  - The winner pays the second highest bid.

Do it?

Play: Ascending Auction (English auction)

- Price starts at reserve price and raises by one cent.
- Buyers indicate their willingness to continue bidding by keeping hands up.
- The auction ends when just one bidder remains.
- The remaining bidder wins and pays the price at which the second remaining bidder dropped out.

Ready?

Canonical Model: The Independent Private Value (IPV) Model

- One unit of indivisible object.
- One seller that gets to set the rules of the auction. Reserve valuation r.
- Potential buyers:
  - N bidders
  - Each bidder i has a (private) value v_i.
  - Each v_i is drawn independently from $\mathcal{U}([0,1])$ (same ideas will apply with different distributions).
  - Bidder i submits bid $b_i(v_1, ..., v_N)$: independence assumption $b_i(v_i)$.
- Information structure: asymmetry of information between seller and bidders.

Canonical Model: The Independent Private Value (IPV) Model

- Notation:
  - $q_i(b_1, ..., b_N)$ is the prob. bidder i gets the object.
  - $p_i(b_1, ..., b_N)$ is the price bidder i pays if she wins the object.
  - Expected utility of bidder $i = q_i * [v_i - p_i] + (1 - q_i) * 0$.
  - Expected utility of seller $= \sum_i q_i * p_i - r$. Normalized to $r = 0$.

First Design: Ascending Auction (English auction)

- Price starts at zero and raises slowly.
- Buyers indicate their willingness to continue bidding (e.g. keep their hands up) or can exit.
- The auction ends when just one bidder remains.
- The remaining bidder wins and pays the price at which the second remaining bidder dropped out.

How should you bid?

Ascending Auction: Bidding

Optimal strategy: Continue bidding until the price just equals your value: $b_i(v_i) = v_i$, for all i.

- This strategy is weakly dominant.
- The bidder with the highest value will win the auction (efficiency).
- The winner will pay the second highest value.

Reminder: Order statistics

- N independent draws from a common distribution F.
- Rank these draws: $w_1^{(N)} \leq w_2^{(N)} \leq ... \leq w_k^{(N)} \leq ... \leq w_N^{(N)}$
- $w_k^{(N)}$ is a random value (k-th order statistic corresponding to N trials) with cdf $F_k^{(N)}$ and pdf $f_k^{(N)}$
  - $F_N^{(N)}(x) = Prob[w_N^{(N)} \leq x] = Prob[max\{v_1, ..., v_N\} \leq x] = Prob[v_1 \leq x \text{ and } ... \text{ and } v_N \leq x] = [F(x)]^N$.
  - $F_{N-1}^{(N)}(x) = Prob[w_{N-1}^{(N)} \leq x] = Prob[\{w_{N-1}^{(N)} \leq x\} \bigcap \{w_N^{(N)} \leq x\}] + Prob[\{w_{N-1}^{(N)} \leq x\} \bigcap \{x \leq w_N^{(N)}\}] = Prob[w_N^{(N)} \leq x] + Prob[w_{N-1}^{(N)} \leq x | x \leq w_N^{(N)}] * Prob[x \leq w_N^{(N)}] = [F(x)]^N + [F(x)]^{N-1} * N * [1 - F(x)] = N * F_N^{(N-1)}(x) - (N-1) F_N^{(N)}(x)$.

Reminder: Uniform case

- Suppose we repeatedly take two independent draws from $\mathcal{U}([0,1])$.
- $F_2^{(2)}(x) = x^2$ so, on average, the highest draw will be $E[w_2^{(2)}] = 2/3$.
- $F_1^{(2)}(x) = 2F_2^{(1)}(x) - F_2^{(2)}(x) = 2x - x^2$ so, on average, the second highest draw will be $E[w_1^{(2)}] = 2E[w_2^{(1)}] - E[w_2^{(2)}] = 1/3$.

Reminder: Uniform case

- Suppose we repeatedly take N independent draws from $\mathcal{U}([0,1])$.
- $F_N^{(N)}(x) = x^N$ and $f_N^{(N)}(x) = Nx^{N-1}$ so, on average, the highest draw will be $E[w_N^{(N)}] = \frac{N}{N+1}$.
- So, on average, the second highest draw will be $E[w_{N-1}^{(N)}] = \frac{N-1}{N+1}$.

Ascending Auction: Revenue

- The average (or expected) revenue of the ascending auction with two bidders whose values are drawn from $\mathcal{U}([0,1])$ is the expected second-highest valuation (second order statistic from 2 draws from F) equal to $1/3$.
- With N bidders with values from $\mathcal{U}([0,1])$:
  - On average, the highest bid (value) will be $\frac{N}{N+1}$.
  - And, on average, the second highest bid (value) will be $\frac{N-1}{N+1}$.
  - So the expected revenue for the seller will be $\frac{N-1}{N+1}$.

Ascending Auction: Profit

- Suppose values are drawn from $\mathcal{U}([0,1])$.
- What profit does a bidder i with value v expect?
  - With 2 bidders i and j, his probability of winning is $Prob[v_j \leq v] = v$.
  - If he wins, he gets an object worth v to him.
  - The expected payment conditional on winning is, with two bidders, $E[v_j | v_j \leq v] = \frac{v}{2}$.
  - So expected profit is $Prob[v_j \leq v](v - E[v_j | v_j \leq v]) = \frac{v^2}{2}$.

Ascending Auction: Profit

- With N bidders, the expected profit of a buyer with value v is:

$$Pr[w_{N-1}^{(N-1)} \leq v] * \left(v - E[w_{N-1}^{(N-1)} | w_{N-1}^{(N-1)} \leq v]\right)$$

$$= v * F_{N-1}^{(N-1)} - \int_0^v F_{N-1}^{N-1}(t)dt = \frac{v^N}{N}$$

Second design: Second Price Auction

- Bidders submit sealed bids.
- The seller opens the bids.
  - The bidder with the highest bid wins the auction.
  - The winner pays the second highest bid.

How should you bid?

Second Price Auction: Bidding

Theorem. It is a weakly dominant strategy to bid your value in a second price auction: $b_i(v_i) = v_i$, for all i.

Proof.

Second Price Auction: Bidding

Theorem. It is a weakly dominant strategy to bid your value in a second price auction: $b_i(v_i) = v_i$, for all i.

Proof.

- Suppose you bid $b > v$.
  - If the highest opposing bid is less than v, or higher than b, it makes no difference.
  - If the highest opposing bid is between v and b, you win if you bid b, but you pay above your value, so better to bid v.
- Suppose you bid $b < v$.
  - It only matters if the highest opposing bid is between b and v. Then bidding v is better because you get the object at a price lower than v.

Second Price Auction: Equilibrium

- Dominance instead of weak dominance if every report is potentially pivotal (see Milgrom's book p. 50).
- In equilibrium, everyone bids their value. Dominant strategy = no matter what others are doing.
- Intuition: aggressive bidding (highest possible bid so as to avoid loss) because your bid does not fix price only winning prob.
- Results:
  - The bidder with the highest value wins the auction.
  - He pays an amount equal to the second highest value.
- It's exactly the same as the ascending auction.
  - Same winner, same revenue, same expected profit for a bidder with value v.

Vickrey Auction

The second price auction is an example of a more general format of auctions, called "Vickrey" auctions, that can be used to sell multiple goods:

- Seller has a set of goods $1, ..., K$ to sell.
- Bidders care about profit equal to value minus payment.
- Bidder i has value $v_i(x)$, defined for any subset of the goods x.

Vickrey Auction: Rules

The rules of a Vickrey auction are:

1. Bidders are asked to submit their (full list of) values.
2. Seller uses the submitted values to find the allocation that leads to the most total surplus where total surplus is measured as the sum of the bidder values.

Vickrey Auction: Rules

3. Seller computes payments. To compute i's payment p_i
   - Let v_i denote the value that i gets from the efficient allocation (if i is assigned a subset of goods x, then $v_i = v_i(x)$).
   - Let V denote the total surplus from the efficient allocation ($V = \sum_i v_i$).
   - Let V^{-i} denote the total surplus that could be generated if i did not participate (and the seller allocated the goods among the other bidders to maximize total value).
   - Let $p_i = v_i - (V - V^{-i})$ be i's Vickrey payment.

Vickrey Auction: Discussion

- Assuming the bidders truthfully reveal their values, the profit that each bidder makes in the auction is exactly equal to the amount by which they increase social surplus. That is, bidder i's profit is $v_i - p_i = V - V^{-i}$.
- If i wasn't around the efficient allocation would create a surplus V^{-i}.
- With i around, the efficient allocation creates surplus V.

$\Rightarrow$ Bidder i gets to keep this difference as profit.

Vickrey Auction: Discussion

- A bidder who wins no items also pays nothing.
- To see this, notice that if the seller assigns the goods efficiently without i being around and when i shows up, it isn't efficient to give her anything, then $V = V^{-i}$, and also $v_i = 0$. So $p_i = 0$.

Vickrey Auction: Discussion

- The optimal strategy for a bidder in a Vickrey auction is to reveal her true value.
- There is a proof of this in the Milgrom's book (p.51), and the logic is similar to the second price auction.

Vickrey Auction: Pay the externality mechanism

In Vickrey auction the prices are set to make the profit of each buyer equal his contribution to total surplus:

$$= \text{Total Surplus if he is counted} - \text{Total Surplus if not counted}$$

- Equivalently, each winner pays the **externality** he imposes by displacing other possible winners.

Vickrey Auction: Pay the externality mechanism

- We then can interpret p_i as having i pay the seller the value she displaces by showing up at the auction and getting her efficient bundle of goods. To see why, notice that $V = \sum_j v_j$. So

$$p_i = v_i - (V - V^{-i}) = v_i - \sum_j v_j + V^{-i} = V^{-i} - \sum_{j \neq i} v_j$$

- Now V^{-i} is the total value from assigning the goods efficiently to all bidders that aren't i, and $\sum_{j \neq i} v_j$ is the total value these bidders get when i shows up and is assigned some of the goods.
- So $V^{-i} - \sum_{j \neq i} v_j$ is the value that the other bidders lose when i shows up and displaces them, and this is also p_i.

How the second price does this?

Single object Vickrey Auction and second price auction

- In the second price auction, bidders are asked to submit their values.
- The high value wins the auction. If the high value is v_1, the total surplus created is $V = v_1$.
- Suppose the next highest value submitted is v_2.

Single object Vickrey Auction and second price auction

- If the high value guy weren't around, the object would go to bidder 2, so $V^{-1} = v_2$.
- Using the Vickrey formula, the winner must pay $p_1 = v_1 - (V - V^{-1}) = v_2$, that is, the second highest stated value.
- So the single object Vickrey auction is just a second price auction!

Third design: First-Price Auction

- The bidders submit sealed bids.
- The seller opens the bids.
  - The bidder who submitted the highest bid wins the auction.
  - The winner pays his own bid.

How should you bid?

First-Price: Bidding

- Best to submit a bid lower than your value. Why? How much less?

First-Price: Bidding

- Best to submit a bid lower than your value. Why? How much less?

  Tradeoff: Submitting a higher bid

  - increases the chance that you win the auction.
  - increases the amount you pay if you win.

- The optimal bid depends on what you think the others will do (unlike the second price auction).
- Hence the need to conduct an equilibrium analysis.

General Intuition

- Suppose all other players use the same increasing strategy $b(v)$.
- Your expected payoff when your value is v_i and your bid is b_i is equal to:

$$U_i = (v_i - b_i) \underbrace{\Pr\left(b_i > max_{j \neq i} b(v_j)\right)}_{\Pr(win) \uparrow b_i}$$

First-Price: Equilibrium

- Suppose the values are drawn independently from $\mathcal{U}([0,1])$.
- Suppose all other bidders use the strategy $b(v) = \alpha v$.
- Then expected payoff is given by

$$\begin{aligned} U_i &= (v_i - b_i) \Pr\left(b_i > max_{j \neq i} b(v_j)\right) \\ &= (v_i - b_i) \Pr\left(b_i > \alpha max_{j \neq i} v_j\right) \\ &= (v_i - b_i) \prod_{j \neq i} \Pr\left(v_j < b_i / \alpha\right) = (v_i - b_i) \left(\frac{b_i}{\alpha}\right)^{N-1} \end{aligned}$$

FOC with respect to b_i for optimal bidding:

$$-\left(\frac{b_i}{\alpha}\right)^{N-1} + (v_i - b_i)(N-1)\frac{1}{\alpha}\left(\frac{b_i}{\alpha}\right)^{N-2} = 0 \;\; \Rightarrow \;\; b_i = \frac{N-1}{N} v_i$$

First-Price: Equilibrium Revenue

- The equilibrium revenue of the seller for $N = 2$ is $E[b(w_2^{(2)})] = \frac{1}{2}E[w_2^{(2)}] = \frac{1}{3}$.
- This is the same revenue as in the ascending and second price auctions!
- With N bidders, the expected revenue of the seller is:
  - The highest bid is $\frac{N-1}{N}v_{max}$.
  - On average $E(v_{max}) = \frac{N}{N+1}$.
  - So the expected revenue is $\frac{N-1}{N+1}$
- Again same revenue as in the ascending and second price auctions!

First-Price: More general form

- Same logic.
- Assumptions: symmetric bidding function (all bidders use same bidding) and monotonicity (bidding function increasing in valuation).
- Optimal (equilibrium) bidding function:

$$b(v) = E[w_{N-1}^{(N-1)} | w_{N-1}^{(N-1)} < v].$$

- See Milgrom's book (p.114).

Fourth: Descending Price Auction (Dutch auction)

- Price starts high.
- Price drops slowly (continuously).
- At any point, a bidder can claim the item at the current price.
- The first one to claim the item wins the auction and pays the current price.

How should you bid?

- This auction is strategically equivalent to the first-price auction
  - Suppose the bidders had to send in computer programs to do the bidding for them... it would be a first price auction!
  - Does actually being there make any difference?
  - No: in both auctions, your bid only matters if you are the winner or tied for winning.
  - You do not learn anything more about the other players by being present in the auction.
  - So for any conjectured strategies of the other bidders, you bid to maximize $\Pr(win)(v - b)$. The same problem as in the first-price auction.
- So the equilibrium strategies are the same as in the first-price auction. And the expected revenue and expected profits are also the same.

Fifth: All Pay Auction

Rules:

- Bidders submit bids.
- Seller opens bids.
- The bidder with the highest bid wins the auction.
- All bidders pay their bids.

How should you bid?

All Pay Auction: Bidding

- Clearly you want to bid less than your value.
- Bidding more means:
  - Greater chance of winning.
  - Pay more for sure (whether you win or you lose).
- Suppose we can find the equilibrium strategies:
  - How will the bids compare to the other auctions?
  - Will the seller raise more or less revenue in equilibrium?

Let's try to solve for the equilibrium bids with N bidders and independent values drawn according to $\mathcal{U}([0,1])$. Assume symmetry and monotonicity of $b(v)$, with $b(0) = 0$.

All Pay Auction: Bidding

- Computation of winning prob. of bidder i: from monotonicity of bidding function the winner is the bidder with highest valuation so $Prob[v_i \geq v_j, \forall j \neq i] = [F(v_i)]^{N-1} = [v_i]^{N-1}$.
- Expected payoff of bidder i: $[F(v_i)]^{N-1} v_i - b(v_i)$.
- Necessary condition for equilibrium is that $b(v_i)$ is optimal that is: $v_i = argmax_z [z]^{N-1} v_i - b(z)$
- FOC: $b'(v_i) = (N-1) v_i^{N-2} v_i = (N-1) v_i^{N-1}$ so $b(v_i) = \frac{N-1}{N} v_i^N$.
- Expected profit of bidder: $\frac{v^N}{N}$.

All Pay Auction: Revenue

- The seller receives $b(v)$ from each bidder with valuation v.
- Therefore, seller's expected revenue is:

$$N \int_0^1 b(v)f(v)dv = \frac{N-1}{N+1}.$$

So same revenue again!

Comparison of Auctions

- We have seen that several auction designs share the following equilibrium properties (at least in our uniform distribution example):
  - The allocation is efficient (highest value wins the auction).
  - Expected revenue is the same.
  - Expected profit of a bidder with value v is the same.
- We will see that this is no accident, and explore this equivalence result further.

Revenue Equivalent Theorem: Background

- We saw that (in theory) some common auction designs all lead to efficient outcomes and yield the same revenue on average.
- Next:
  - How general is this result?
  - Why is this the case?
  - What are the implications?

Revenue Equivalence Theorem (RET)

Theorem: Suppose N bidders have values $v_1, ..., v_N$ identically and independently distributed with cdf F. Then all auction mechanisms that (i) always award the object to the bidder with highest value in equilibrium, and (ii) give a bidder with the lowest valuation (here 0) zero profits, generates the same revenue in expectation.

Understanding the Math: Envelope Theorem

- Consider a parameterized maximization problem:

$$U(v) = max_b \, u(b, v) = u\big(b^*(v), v\big)$$

- By the FOC, maximization means that when $b = b^*(v)$:

$$u_b(b, v) = 0$$

- By the chain rule for differentiation:

$$U'(v) = \underbrace{u_b\big(b^*(v), v\big)}_{=0} b^{*\prime}(v) + u_v\big(b^*(v), v\big)$$

and the envelope theorem just says this:

$$U'(v) = u_v\big(b^*(v), v\big)$$

Proof of the RET

In any auction format, a bidder has to submit a "bid" b (an actual bid, or a stopping price).

In equilibrium, the bid of the bidder solves:

$$U(v) = max_b v \Pr(Win \mid b) - E(Payment \mid b)$$

By the envelope theorem:

$$U'(v) = \Pr\big(Win \mid b^*(v)\big)$$

So the bidder's expected profit is:

$$U(v) = U(0) + \int_0^v \Pr\big(Win \mid b^*(v)\big) dv$$

Proof of the RET

In any auction format that satisfies the conditions of the theorem, we have $U(0) = 0$.

Therefore the expected profit of a bidder with value v is

$$U(v) = \int_0^v \Pr\big(Win \mid b^*(v)\big) dv$$

But by the conditions of the theorem, the equilibrium probability of winning is the probability of having the highest value:

$$\Pr\big(Win \mid b^*(v)\big) = \Pr(v = v_{max})$$

Proof of the RET

So for all these auction formats, we get the same expected revenue for a bidder with value v.

Therefore the expected revenue of the seller is also the same:

$$Expected\ Revenue = \underbrace{Total\ Surplus}_{Ev_{max}} - N * (Expected\ Profit)$$

RET: Solving the First-Price Auction

- Two bidders and values $\mathcal{U}([0,1])$.
- In the second-price auction, a bidder with value v will win with probability v and expects to pay $v/2$ when he wins. So the expected profit of a player with value v is:

$$U_{SPA}(v) = v^2/2$$

- Suppose the first-price auction has a symmetric equilibrium $b(v)$ where $b(v)$ is strictly increasing. Then the highest value bidder wins so the RET must hold in this equilibrium.
- Now the expected profit of a bidder with value v in the FPA is

$$U_{FPA}(v) = v\big(v - b(v)\big)$$

- By the RET, we have $U_{FPA}(v) = U_{SPA}(v) \quad \Rightarrow \quad b(v) = v/2$

RET: All-Pay Auction

- Two bidders and values $\mathcal{U}([0,1])$.
- Suppose the all-pay auction has a symmetric equilibrium $b(v)$ where $b(v)$ is strictly increasing. Then the highest value bidder wins so the RET must hold in this equilibrium.
- Now the expected profit of a bidder with value v in the APA is

$$U_{APA}(v) = v^2 - b(v)$$

- By the RET, we have

$$U_{APA}(v) = U_{SPA}(v) \quad \Rightarrow \quad b(v) = v^2/2$$

“Hidden” Assumptions

- Bidders know their own value
  - What if they may want to resell the good?
  - What if another bidder has relevant information?
- Bidder values are independent
  - Then bidder i and j will have the same belief about k's value.
  - What if the item for sale is an oil field, and the bidders conduct surveys of the field?
- Bidders care about their profit in the auction
  - What if they care about how much others pay?
  - What if they are risk averse?

RET as a Benchmark

- RET is a central result in auction theory, but it depends on strong assumptions.
- Many results in economics are similar in their dependence on implausible assumptions.
  - First and second welfare theorems.
  - Coase theorem.
- What is the significance of this kind of results?
  - Structures our way of thinking and gives research directions: if we think the conclusion does not hold in reality, which assumption fails...

Are bidders really strategic?

- So far we've assumed that bidders understand their environment and behave **strategically**. A good assumption?
- No simple answer to this question:
  - Lots of evidence against the assumption when looking at bidders in novel environments with little feedback.
  - But potentially a reasonable assumption for experienced and sophisticated bidders in familiar environments.

Excess bidding on eBay?

- Lee and Malmendier (2011):
  - Auctions for a board game, in 2004
  - Available from two eBay sellers at posted price: $129.95.
  - Price went higher in 42 % of the auctions.
- Einav, Kuchler, Levin and Sundaresan (2015):
  - Hundreds of thousands of cases where an eBay seller sold a product by auction while also selling it at a posted price.
  - Auction prices exceed posted prices fairly often in 2003, but by 2009 in relatively few auctions, and usually by just a few dollars.

Auction design

- In the IPV case, all standard auctions are equivalent for revenues and bidder expected profit. So what can be designed?
  - Add reserve price: a price below which the seller will not sell.
  - Add subsidies: subsidize the cost of participating in the auction for some potential bidders.
- There are also some practical concerns such as, how to avoid collusion or favor entry.

Reserve price: example

- 2 bidders with values $v_1, v_2 \sim \mathcal{U}\big([0,1]\big)$.
- We saw that the expected revenue is $1/3$.
- Can the seller benefit from a reserve price?

Reserve price

- The seller sets a reserve price R below which he will not sell.
- Second price auction:
  - A bidder with value $v \leq R$ will not bid.
  - A bidder with value $v > R$ bids her value.
- Then this bidder expects to win with probability v.
- Conditional on winning: pay either R or opponent value.
- Expected profit of bidder with value $v > R$:

$$v^2 - v\left(\frac{R}{v} \times R + \frac{v-R}{v} \times \frac{v+R}{2}\right)$$

Reserve price

- Is it worth setting up a reserve price?
- Expected revenue with reserve price$= 2\left(\frac{R^2}{2} - \frac{2R^3}{3} + \frac{1}{6}\right)$.
- Optimal reserve price $R^* = \frac{1}{2}$.
- More general formula for N bidders, expected revenue

$$N\left(\frac{R^N}{N} - \frac{2R^{N+1}}{N+1} + \frac{N-1}{N(N+1)}\right).$$

- With uniform independent private values $R^* = 1/2$, independently of N.

Optimal Auctions Theory

- A more general version of the RET says that for any auction, the expected revenue of the seller will be equal to the expected "marginal revenue" from the winning bidder.

- Then, the optimal auction should give the object to the bidder with the highest marginal revenue as long as it is positive.

We can always write with n bidders:

$$\mathbf{R} = \int_{\underline{v}_1}^{\overline{v}_1} \cdots \int_{\underline{v}_n}^{\overline{v}_n} \sum_{i=1}^{n} g_i(v_i) \Pr\big(i\ wins \mid v_1, \cdots, v_n\big) dv_1 \cdots dv_n - \sum_{i=1}^{n} U_i(0)$$

where $g_i(v_i)$ is the "marginal revenue" function.

Note that the probability has been changed to depend only on v. This is because the auction can always be reinterpreted as a mechanism in which bidders are asked to announce their values.

Note that the marginal revenue function depends only on the distribution of values.

Optimal Auctions

Theorem (Myerson): If for every bidder, the function $g_i(v_i)$ is increasing, the following auction (mechanism) is incentive compatible (that is bidders will reveal their true value) and optimal:

- Bidders report their value to the seller.
- The bidder with the highest "marginal revenue" $g_i(v_i)$ wins the auction, provided $g_i(v_i) > 0$.
- The winner pays an amount equal to the least value he could have reported and still won (that is the value of the bidder with the second highest marginal revenue).

Optimal Auctions

Remarks:

- If bidders are symmetric, the optimal auction is a standard auction with reserve price.
- If asymmetric, the optimal auction biases the allocation toward bidders with high marginal revenues, but perhaps lower values.

Example with Asymmetric Bidders

- Two bidders with values:
  - Bidder 1: $\mathcal{U}([20, 60])$.
  - Bidder 2: $\mathcal{U}([0, 80])$.
- What are the marginal revenues?
  - Bidder 1: The price corresponds to v. The quantity corresponds to the probability of buying at price v so $q = \frac{1}{40}(60 - v)$ and $v = 60 - 40q$:

$$g_1(v_1) = \frac{d((60 - 40q)q)}{dq} = 60 - 80q = 2v_1 - 60.$$

  - Similarly for Bidder 2:

$$g_2(v_2) = 2v_2 - 80.$$

Bidder 1 could have a higher marginal revenue with a lower value.

The optimal auction is going to be biased toward him.

Subsidies and set-asides

- In real world auctions, it is common to see sellers choose to treat some bidders preferentially. Why?
  - For distributional reasons: e.g. state and federal procurement explicitly favors domestic, small or minority-owned businesses by restricting entry or giving subsidies.
  - For competition reasons: to “level” the playing field, or encourage entry and create competition.

Subsidies and set-asides

- In the U.S., under the Buy-American Act, the US Government offers a 6 percent bid preference to domestic suppliers.
- European governments do not explicitly state the formulae by which local and small bids are to be compared with foreign bids: governments achieve favoritism by more covert methods

E.g. short time for the submission of bids, residence requirements on bidders, defining technical requirements, etc. See Coviello and Mariniello (2013).

Raising revenue at the FCC

- Ian Ayres and Peter Cramton have argued that the FCC's policy of subsidizing minority-owned firms raised revenue when the FCC started selling spectrum licenses in the 1990s.
- How? These firms pushed the large bidders to bid higher, raising the final auction prices.

FCC and competition policy:

- Suppose the FCC plans to allocate 7 national licenses and bidders want to buy multiple licenses.
- How to ensure that there are more than two major players in the industry after the auction?
  - Give “smaller” carriers an x % discount?
  - Restrict each carrier to no more than x licenses?
  - Restrict largest carriers to x licenses in total?
- Problem: how to choose x in each case! Set-asides provide more certainty about who wins, but might end up with lower revenue.

Practical issues

- Our analysis has assumed:
  - A fixed set of bidders willing to participate
  - Bidders behave competitively, or "non-cooperatively"
- Practical auction design has to worry about
  - Collusion - bidders may cooperate not compete
  - Entry - can be hard to get bidders to participate

Collusion

- Collusion occurs if bidders agree in advance or during the auction to let prices settle at a low level.
  - This is generally illegal, but it can and does happen.
- Concern is often biggest (and can be less obviously illegal) when there are multiple items for sale: e.g. "you take these, I'll take these ? Let's end the auction".
- With a single item, collusion might rely on
  - Side agreements: you win today, and share the profit with me.
  - Intertemporal trades: you win today, I'll win tomorrow.

Collusion

- Collusion among bidders is certainly more likely if the number of participants is small

  In Public-Private Partnerships (PPP) in the United Kingdom, there is an average of four bidders per PPP contract.

  In France for water concession average of five bidders.

Deterring collusion

- What auction design works to deter collusion?
- Ascending:
  - Suppose bidders A and B agree in advance that A should win at a low price. Although B can deviate from the agreement and keep bidding, A can just bid back - helps enforce the agreement.
- Sealed bidding:
  - If A and B agree that A should win at a low price, A must submit a low bid. But then B can send in a slightly higher bid and win the auction at a low price. Makes it harder to sustain collusion.

Entry

- A typical problem in organizing auction sales is to make sure that enough bidders will participate
- Auctions rely on having competition to set the price, and bidders may not want to participate unless they think they have a good chance.
- When might entry be a concern, and what can be done?

Entry example

- Two bidders, values drawn from $\mathcal{U}\left([0,100]\right)$ and $\mathcal{U}\left([0,200]\right)$.
- Bidder expected profits from ascending auction
  - Bidder 1 expects to win with probability 1/4. When he does win, he expects to have a value of 66 and to pay 33. So his overall expected profit if he participates is 8.3
  - Bidder 2 expects with probability 1/2 to have a value above 100, and in this case make 150-50=100, or a value below 100, in which case he expects to make (1/2)*(66-33)=16.6. So his overall expected profit is 58.3
- If the cost of entry is 10, Bidder 1 won't even bother.
- And Bidder 2 could end up winning at a price of zero.

Auction design to promote entry

- **Subsidize weaker bidders**
  - Encourages their participation - can also restrict very strong bidders from entering (a "set-aside" policy)
- **Subsidize entry costs directly**
  - E.g. in architectural competitions, architects partly reimbursed for building a model needed to submit a bid for the contract.
- **Sealed bidding**
  - Prospect of very low prices in sealed bid auction encourages bidders to enter and try to "steal" the auction - whereas with open bidding, a strong bidder can respond.

Auction design to promote entry

- Careful with design: Marion (2007) and Krasnokutskaya and Seim (2010) study the effect of small business bid subsidies in California highway procurement auctions, and Athey, Coey and Levin (2013) analyze set-asides of forest service timber auctions for small businesses in the US.

  Find that discriminatory programs have negative effects on public budgeting, increasing procurement costs and reducing government revenue.

  Channel: the pool of efficient firms participating to the procurement contest seems to be determinant for procurement costs! Dynamic aspects.

Summary

- Auction design can involve multiple objectives:
  - Efficiency: making sure high value bidder wins.
  - Revenue: getting the best possible return on the sale.
  - Distributional: ensuring that certain bidders have a chance.
- There are often trade-offs between these objectives
  - Reserve prices and subsidies can sometimes increase revenue even though they may distort the auction away from efficiency.
- Practical auction design also has to worry about basic economic issues such as collusion and entry.
- Practical auction design also has to consider economics after the auction (e.g. competition in the mobile industry, resale).

Outline of the class

Lecture 1: Introduction

Lecture 2: Auction theory and design

Lecture 3: Common-value and Multi-unit auctions

Lecture 4: Multi-item auctions and matching, sponsored-search auctions, spectrum auctions, package auctions