Supplementary Material for

"Discrimination in Dynamic Procurement Design with Learning-by-doing"

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This is the supplementary material for the paper entitled "Discrimination in Dynamic Procurement Design with Learning-by-doing." It contains: (i) the extension of our simple model to the case where one local firm and many global firms compete for the public good provision, (ii) the characterization of optimal procurement mechanism with commitment, (iii) characterization of the optimal dynamic procurement mechanism when a global firm has synergies disadvantages, and (iv) the full characterization of the optimal second-period mechanism.

S.1 Discrimination in Dynamic Procurement Design with Learning-by-doing: The Case with Many Global Firms

This section contains the complete treatment of the results described in Section 7.1. We consider the extension of our simple model to the case where one local firm and many global firms compete for the public good provision.

We now suppose that in the city-economy there are one local L and N global G firms with $N > 1$. As in Section 3 of the paper, we assume that there is learning-by-doing such that the incumbent firm, either a local or a global firm, becomes a *strong* one in period 2. As in the simple model, global firms may have access to learning-by-doing even if they are not the incumbent firm in the local city. For simplicity, we assume that global firms' transferability is identically and independently drawn from a Bernoulli distribution with parameter θ such that with probability θ (resp. $1 - \theta$), an entrant global firm is a *strong* (resp. *weak*) one in period 2.

The timing of the game is identical to the one described in the paper.

In the second period, depending on the selection of the first-period provider and on the realization of transferability, there exist many possible contingencies. As in the paper, all possible contingencies are summarized by a state variable X. A state X in period 2 depends on the status of the local and global firms at date (vii), either strong or weak. To derive our results we do not have to be explicit about the number of states.

The *optimal sequential direct mechanism* is defined as $\mathcal{M}_1 = (\Delta_1, p_1, T_1)$, the first-period direct mechanism, and $\mathcal{M}_2(X) = (\Delta_2(X), p_2(X), T_2(X))$, the second-period direct mechanism in each possible contingency X in period 2, where $\Delta_t = (\Delta_{Lt}, \Delta_{G_1t}, ..., \Delta_{G_Nt})$ is the set of possible costs for each firm in t; $p_t = (p_{Lt}, p_{G_1t}, ..., p_{G_Nt})$ is an *allocation rule* such that $p_t : \Delta_t \to P$, where P is the set of probability distributions over the set of firms; and $T_t = (T_{Lt}, T_{G_1t}, ..., T_{G_Nt})$ is a *payment rule* such that $T_t : \Delta_t \to \mathbb{R}^{N+1}$.

The direct mechanism $\mathcal{M}_t$ in each period t maximizes the total social welfare net of the cost of public funds, subject to three constraints: individual rationality constraints in t, incentive compatibility constraints in t, and possibility constraints in t. For all firms' revealed cost in period t, the allocation and payment rules determine the probability $p_{it}(c_t)$ that the firm $i \in \{L, G_1, .., G_N\}$ will provide the public good in period t, and the expected monetary transfer $T_{it}(c_t)$ that i will receive at period t, where $c_t = (c_{Lt}, c_{G_1t}, ..., c_{G_Nt})$ is the vector of firms' cost in period t.

As in the basic model, in order to characterize the optimal sequential mechanism, we solve the model by backward induction. We first find the optimal second-period mechanism in each possible contingency in period 2. Then, we characterize the optimal first-period mechanism, which takes into account the optimal mechanism chosen in period 2.

S.1.1 Second-period Optimal Mechanism

The public authority designs a direct mechanism $\mathcal{M}_2(X)$ that maximizes

$$W_2(X) = \int_{\Delta c_2(X)} \sum_{i=L,G_1,...,G_N} \Big\{ \big(p_{i2}(c_2,X)\big)S + \alpha\big(T_{i2}(c_2,X) - c_{i2}p_{i2}(c_2,X)\big) \\ -(1+\lambda)T_{i2}(c_2,X)\Big\} f_2(c_2|X)dc_2 \qquad (P_{NI})$$

subject to the following constraints:

1. individual rationality constraints in state X:

$$U_{i2}(c_{i2},X) \geq 0, \forall i, \forall c_{i2} \in \Delta_{i2}(X); \qquad (IR_2(X))$$

2. incentive compatibility constraints in state X:

$$U_{i2}(c_{i2},X) = U_{i2}(c_{i2},c_{i2},X) \geq U_{i2}(\hat{c}_{i2},c_{i2},X), \forall i, \forall c_{i2}, \hat{c}_{i2} \in \Delta_{i2}(X), \qquad (IC_2(X))$$

   where $U_{i2}(\hat{c}_{i2},c_{i2},X) = E_{c_{-i2}}[T_{i2}(\hat{c}_{i2},c_{-i2},X) - c_{i2}p_{i2}(\hat{c}_{i2},c_{-i2},X)|X]$;

3. possibility constraints in state X:

$$p_{i2}(c_2,X) \geq 0 \text{ and } \sum_i p_{i2}(c_2,X) = 1, \forall c_2 \in \Delta_2(X), \forall i, \qquad (PC_2(X))$$

where $i = \{L, G_1, ..., G_N\}$, $\Delta_2(X) = \Delta_{i2}(X) \times \Delta_{-i2}(X)$, and
$f_2(c_2|X) = f_{c_{L2}}(c_{L2}|X)f_{c_{G_1 2}}(c_{G_1 2}|X)...f_{c_{G_N 2}}(c_{G_N 2}|X)$.

By the Envelope Theorem,

$$\frac{dU_{i2}}{dc_{i2}} = -E_{c_{-i2}}[p_{i2}(c_{i2},c_{-i2},X)|X] = -Q_{i2}(c_{i2},X). \qquad \text{(S.1)}$$

Equation (S.1) is a local incentive condition. It is a necessary and sufficient condition if the following condition holds:

$$\frac{dQ_{i2}(c_{i2},X)}{dc_{i2}} \leq 0. \qquad \text{(S.2)}$$

Integrating equation (S.1), we have that

$$U_{i2}(c_{i2},X) = U_{i2}(\overline{c_{i2}}(X),X) + \int_{c_{i2}}^{\overline{c_{i2}}(X)} Q_{i2}(s_{i2},X)ds_{i2}. \tag{S.3}$$

Standard treatment of this problem implies that the public authority's problem P_{NI} can be written as:[1]

$$W_2(X) = \int_{\Delta c_2(X)} \Big\{\Sigma_i\Big(S-(1+\lambda)c_{i2}-(1+\lambda-\alpha)\frac{F_{c_{i2}(c_{i2},X)}}{f_{c_{i2}(c_{i2},X)}}\Big)p_{i2}(c_2,X)\Big\}f_2(c_2|X)dc_2$$

$$-(1+\lambda-\alpha)(\Sigma_i U_{i2}(\bar{c}_{i2}(X),X))$$

Consequently, local authority problem is to maximize $W_2(X)$ subject to

$U_{i2}(\overline{c_{i2}}(X),X) \geq 0, \forall i;$

$\frac{dQ_{i2}(c_{i2},X)}{dc_{i2}} \leq 0, \forall i;$

$p_{i2}(c_2,X) \geq 0$, $\forall i$, and $\sum_i p_{i2}(c_2,X) = 1$.

The following proposition characterizes the optimal mechanism at each state X in the second period.

Proposition S.1 *The optimal second-period mechanism in state X satisfies: $\forall i$, $U_{i2}(\overline{c_{i2}}(X),X) = 0$ and $p_{i2}(c_2,X) = 1$ if and only if*

$$(1+\lambda)c_{i2} + (1+\lambda-\alpha)\frac{F_{c_{i2}}(c_{i2},X)}{f_{c_{i2}}(c_{i2},X)}$$

$$= \min\left\{(1+\lambda)c_{j2} + (1+\lambda-\alpha)\frac{F_{c_{j2}}(c_{j2},X)}{f_{c_{j2}}(c_{j2},X)}, \forall j \in \{L, G_1, ..., G_N\}\right\}; \tag{S.4}$$

otherwise $p_{i2}(c_2,X) = 0$.

Proof S.I *The proof is similar to those of Myerson (1981), McAfee and McMillan (1989), and Naegelen and Mougeot (1998) for the case with two firms.* □

The main difference with the Proposition 1 of the paper is that the public authority now compares the virtual costs of many firms. Therefore, the winning firm is the one with the lowest virtual production cost among the set of $N+1$ firms.

[1] It follows the same approach used to solve the public authority's second-period problem P_I in Section 4 in the paper.

S.1.2 First-period procurement

S.1.2.1 Continuation Payoffs

The continuation payoffs are computed at the end of the first period, after first-period public good provision was awarded and before Nature draws the transferability. That corresponds to the end of date (v) in the Timing. In period 1, neither the authority nor the firms know firms' second-period costs and global firm's transferability. However, as the public authority will optimally select and pay firms in period 2 according to the second-period mechanism described in Proposition S.1, we can compute firms' expected equilibrium payoff and public authority's expected equilibrium payoff at the beginning of period 2.

In order to characterize the continuation payoffs we proceed in three steps. First, we introduce some additional notations and derive intermediate results. Then, we characterize firms and public authority's payoffs after global firms' transferability is realized, but before second-period costs are draw. Finally, we turn to the characterization of firms' and public authority continuation payoffs before transferability is drawn. This last step corresponds to Lemmas 1 and 2 in the paper.

Preliminaries. To compute the ex-ante payoff of a firm i in period 2 what matters is the type of the firm, either *strong* or *weak*, and the number of its strong and weak opponents in period 2. For this reason, let us introduce the following notation.

Definition 1 *We define $v(h_i, N+1-k, c_i)$ as the probability, computed at date $(viii)$, of a firm i of type $h \in \{s, w\}$ with second-period cost c_i to be the supplier in the second-period according to the mechanism described in Proposition S.1, where k is the total number of weak firms in period 2.*

Note that depending on the state in period 2, $N+1-k$ can vary from 1 to $N+1$, equivalently k can vary from 0 to N. We have that $N+1-k$ is equal to 1 when only the incumbent firm (local or global) is a strong firm in the second-period. Yet $N+1-k = N+1$, or $k = 0$, when the incumbent is a local firm together with N global entrant firms with transferability such that there are $N+1$ strong firms in period 2. In order to simplify the notation we order the firms so that the first k firms are weak and the remaining firms are strong.

According to Proposition S.1 and Definition 1, we have for all i

$$v(s_i, N+1-k, c_i) = \int_{\Delta c} \text{Prob}\Bigg[(1+\lambda)c_i + (1+\lambda-\alpha)\frac{F_s(c_i)}{f_s(c_i)} \leq$$

$$\min\left\{\{(1{+}\lambda)c_j{+}(1{+}\lambda{-}\alpha)\frac{F_w(c_j)}{f_w(c_j)},\forall j=1,...,k\},\{(1{+}\lambda)c_j{+}(1{+}\lambda{-}\alpha)\frac{F_s(c_j)}{f_s(c_j)},\forall j=k{+}1,...,N\}\right\}\Bigg]f(c)dc, \tag{S.5}$$

where $\Delta c = \underbrace{\Delta_w \times ... \times \Delta_w}_{k} \underbrace{\Delta_s \times ... \times \Delta_s}_{N-k}$, $f(c) = f_w(c_1)...f_w(c_k)f_s(c_{k+1})...f_s(c_N)$, and $c = (c_1, ..., c_N)$.

Note that $v(s_i, N+1-k, c_i)$ corresponds to $Q_{i2}(c_{i2}, X)$ in equation (S.1), where in the latter X summarizes the states in the second-period; whereas in the $v(h_i, N+1-k, c_i)$ the states are characterized by number of strong $(N-k+1)$ and weak (k) opponents in period 2.

We use the fact that costs are independently and identically distributed across firms and after manipulating equation (S.5) we obtain that

$$v(s_i,N{+}1{-}k,c_i) = \int_{\Delta\tilde{c}} \text{Prob}\left[(1{+}\lambda)c_i{+}(1{+}\lambda{-}\alpha)\frac{F_s(c_i)}{f_s(c_i)} \leq (1{+}\lambda)c_j{+}(1{+}\lambda{-}\alpha)\frac{F_s(c_j)}{f_s(c_j)}\right]^{N-k} \times$$

$$\text{Prob}\left[(1+\lambda)c_i + (1+\lambda-\alpha)\frac{F_s(c_i)}{f_s(c_i)} \leq (1+\lambda)c_z + (1+\lambda-\alpha)\frac{F_w(c_z)}{f_w(c_z)}\right]^k f(\tilde{c})d\tilde{c},$$

where $\Delta\tilde{c} = \Delta_s \times \Delta_w$, $f(\tilde{c}) = f_s(c_j)f_w(c_z)$, and $\tilde{c} = (c_j, c_z)$.

We define $\Phi(c_j) = (1{+}\lambda)c_j{+}(1{+}\lambda{-}\alpha)\frac{F_h(c_j)}{f_h(c_j)}$, where $h = \{s, w\}$. Notice that, for all j, $\Phi_j(c_j)$ is positive and monotonically increasing in c_j. Because c_j is distributed according to F_s, then there exists a distribution $\hat{F}_s$ according to which $\Phi(c_j)$ is distributed. Equivalently, because c_z is distributed according to F_w, then there exists a distribution $\hat{F}_w$ according to which $\Phi(c_z)$ is distributed. Consequently, we have that, given c_i, $\text{Prob}\Big[\Phi(c_i) \leq \Phi(c_j)\Big] = 1 - \hat{F}_s(\Phi(c_i))$, and $\text{Prob}\Big[\Phi(c_i) \leq \Phi(c_z)\Big] = 1 - \hat{F}_w(\Phi(c_i))$. The following Lemma characterizes $\hat{F}_s$ and $\hat{F}_w$ and displays its properties.

Lemma S.1 *The distribution functions of the random variables $\Phi(c_j)$ and $\Phi(c_z)$ are defined respectively as $\hat{F}_s$ and $\hat{F}_w$ with*

(i) $\hat{F}_s(c)$ is equal to $F_s(\Phi_j^{-1}(c))$, with pdf $\hat{f}_s(c) = f_s(c)\frac{d\Phi_j^{-1}(c)}{dc}$, where $\Phi_j^{-1}(c)$ is the inverse function of $\Phi_j(c)$;

(ii) $\hat{F}_w(c)$ is equal to $F_w(\Phi_z^{-1}(c))$, with pdf $\hat{f}_w(c) = f_w(c)\frac{d\Phi_z^{-1}(c)}{dc}$, where $\Phi_z^{-1}(c)$ is the inverse function of $\Phi_z(c)$;

(iii) Additionally, the following inequality holds

$$\hat{F}_w(c) > \hat{F}_s(c). \tag{S.6}$$

Proof S.II *We prove Lemma S.1 by parts.*

Proof Lemma S.1 (i): Define $\Phi_j(C) = (1+\lambda)C + (1+\lambda-\alpha)\frac{F_s(C)}{f_s(C)}$, *with* C *distributed according to* F_s. *So,* $\hat{F}_s(c) = Prob((1+\lambda)C + (1+\lambda-\alpha)\frac{F_s(C)}{f_s(C)} \leq c) = Prob(\Phi_j(C) \leq c) = Prob(C \leq \Phi_j^{-1}(c)) = F_s(\Phi_j^{-1}(c))$. *So,* $\hat{f}_s(c) = \frac{d\hat{F}_s(c)}{dc}$ *and it follows that* $\hat{f}_s(c) = f_s(c)\frac{d\Phi_j^{-1}(c)}{dc}$.

Proof Lemma S.1 (ii): Similar to Proof of Lemma S.1 (i).

Proof Lemma S.1 (iii): Define the random variables $Y = \Phi_s(C) = (1+\lambda)C + (1+\lambda-\alpha)\frac{F_s(C)}{f_s(C)}$, *with distribution* $\hat{F}_s$ *and* $Z = \Phi_w(C) = (1+\lambda)C + (1+\lambda-\alpha)\frac{F_w(C)}{f_w(C)}$, *with distribution* $\hat{F}_w$. *As* $\Phi_h(C) \geq 0$ *and* $\frac{\Phi_h(C)}{dC} > 0$, *the function* $\Phi_h(C)$ *is invertible for any* $h = \{s, w\}$. *By definition of* $\hat{F}_s(c)$ *and invertibility of* $\Phi_s(C)$, $\hat{F}_s(c) = Prob[Y \leq c] = Prob[\Phi_s(C) \leq c] = Prob[C \leq \Phi_s^{-1}(c)]$. *Note that* $\Phi_s(C) > \Phi_w(C)$, *because* $\frac{F_s(c)}{f_s(c)} > \frac{F_w(c)}{f_w(c)}$. *Consequently,* $\Phi_s^{-1}(C) < \Phi_w^{-1}(C)$. *Therefore,* $Prob[C \leq \Phi_s^{-1}(c)] < Prob[C \leq \Phi_w^{-1}(c)] = Prob[\Phi_w(C) \leq c] = Prob[Z \leq c] = \hat{F}_w(c)$. *Hence,* $\hat{F}_s(c) < \hat{F}_w(c)$.

□

Given Lemma S.1 and the fact that costs are independently and identically distributed across firms, we obtain that

$$v(s_i, N+1-k, c_i) = [1 - \hat{F}_s(\Phi_s(c_i))]^{N-k}[1 - \hat{F}_w(\Phi_s(c_i))]^k. \tag{S.7}$$

Similarly,

$$v(w_i, N+1-k, c_i) = [1 - \hat{F}_s(\Phi_w(c_i))]^{N+1-k}[1 - \hat{F}_w(\Phi_w(c_i))]^{k-1}. \tag{S.8}$$

The following Lemma displays some properties of the function $v(h_i, N+1-k, c_i)$.

Lemma S.2 *The function* $v(h_i, N+1-k, c_i)$ *has the following properties:*

(I) The function $v(h_i, N+1-k, c_i)$ *with* $h = \{s, w\}$, *is the same for all* i.

(II) $\lim_{N\to+\infty} v(h_i, N+1-k, c_i) = 0$, *for* $\forall h$, $\forall i$ *and for any* $k \in \{0, ..., N\}$;

(III) $\frac{\partial v(h_i, N+1-k, c_i)}{\partial N} < 0$, *for* $\forall h$, $\forall i$ *and for any* $k \in \{0, ..., N\}$;

(IV) $\lim_{N\to+\infty} \frac{\partial v(h_i, N+1-k, c_i)}{\partial N} = 0$, *for* $\forall h$, $\forall i$ *and for any* $k \in \{0, ..., N\}$;

Proof S.III *We prove Lemma S.2 by parts.*

Lemma S.2 (I) follows from the fact all firms are symmetric.

Lemma S.2 (II): As $1-\hat{F}_h(\Phi_j(c_i)) \in (0,1)$*, for* $h=\{s,w\}$ *and* $j=\{s,w\}$*, then it follows that*

$$\lim_{N\to+\infty} v(h_i, N+1-k, c_i) = 0.$$

Lemma S.2 (III): We have

$$\frac{\partial v(s_i, N+1-k, c_i)}{\partial N} = [1-\hat{F}_s(\Phi_s(c_i))]^{N-k}[1-\hat{F}_w(\Phi_s(c_i))]^k \ln\Big(1-\hat{F}_s(\Phi(c_i))\Big).$$

Because $1-\hat{F}_s(\Phi_s(c_i)) \in (0,1)$*, then* $\ln\Big(1-\hat{F}_s(\Phi_s(c_i))\Big) < 0$*. So, it follows that* $\frac{\partial v(s_i,N+1-k,c_i)}{\partial N} < 0$*. Similarly,* $\frac{\partial v(w_i,N+1-k,c_i)}{\partial N} < 0$*.*

Lemma S.2 (IV): $\lim_{N\to+\infty} \frac{\partial v(s_i,N+1-k,c_i)}{\partial N}$ *is equal to*

$$\ln\Big(1-\hat{F}_s(\Phi_s(c_i))\Big) \lim_{N\to+\infty} \Big\{[1-\hat{F}_w(\Phi_s(c_i))]^k[1-\hat{F}_s(\Phi_s(c_i))]^{N-k}\Big\}$$

Because $1-\hat{F}_s(\Phi_s(c_i)) \in (0,1)$*, then*

$$\lim_{N\to+\infty} \Big\{[1-\hat{F}_w(\Phi_s(c_i))]^k[1-\hat{F}_s(\Phi_s(c_i))]^{N-k}\Big\} = 0, \forall k.$$

So, it follows that $\lim_{N\to+\infty} \frac{\partial v(s_i,N+1-k,c_i)}{\partial N} = 0$*. Similarly,* $\lim_{N\to+\infty} \frac{\partial v(w_i,N+1-k,c_i)}{\partial N} = 0$*.*

□

Having introduced some notations and derived intermediate results, let us characterize the continuation payoffs.

Firms' continuation payoff: After transferability, before second-period costs are drawn. By Proposition S.1 and equation (S.3), we have that firm i expected profits at state X in period 2 is given by

$$U_{i2}(c_{i2}, X) = \int_{c_{i2}}^{\overline{c_{i2}}(X)} Q_{i2}(s_{i2}, X) ds_{i2}.$$

In the notation that the states are characterized by number of strong and weak opponents in period 2, firm i expected profits at state with $N-k+1$ strong and k weak firms in period 2 is given by

$$U_2^i(h_i, N+1-k, c_i) = \int_{c_i}^{\bar{c}_h} v(h_i, N+1-k, s_i) ds_i, \quad \forall h = \{s, w\}, \forall i. \tag{S.9}$$

Hence, before knowing its second-period cost, the ex-ante second-period expected payoff of a firm i in a state in which it is $h = \{s, w\}$, and there are $N+1-k$ strong firms is

$$\begin{aligned} \widetilde{U}(h, N+1-k) \equiv \widetilde{U}_2^i(h_i, N+1-k) &= \int_{\underline{c}_h}^{\bar{c}_h} \Big[\int_{c_i}^{\bar{c}_h} v(h_i, N+1-k, s_i) ds_i \Big] f_h(c_i) dc_i \\ &= \int_{\underline{c}_h}^{\bar{c}_h} v(h_i, N+1-k, c) F_h(c) dc, \quad \forall h, \forall i... \end{aligned} \tag{S.10}$$

Firms' continuation payoff: Before transferability. Having described firms' payoff after transferability, let us turn to the characterization of firms' continuation payoff. The following Lemma summarizes firms' continuation payoff.

Lemma S.3 *The continuation payoff $U_i^C(p_{L1}, p_{G_1 1}, ..., p_{G_N 1})$ of firm i is such that*

(i) if the local firm wins the first-period public good provision, i.e. $p_{L1} = 1$ and $p_{G_i 1} = 0$ for all i, then

$$U_L^C(1, 0, ..., 0) = \sum_{k=0}^{N} \binom{N}{k} \theta^{N-k} (1-\theta)^k \widetilde{U}(s, N+1-k),$$

$$U_{G_i}^C(1, 0, ..., 0) = \theta \Big[\sum_{k=0}^{N-1} \binom{N-1}{k} \theta^{N-k-1} (1-\theta)^k \widetilde{U}(s, N+1-k) \Big]$$

$$+(1-\theta) \Big[\sum_{k=1}^{N} \binom{N}{k} \theta^{N-k} (1-\theta)^k \widetilde{U}(w, N+1-k) \Big];$$

(ii) if a global firm i wins the first-period public good provision, i.e. $p_{G_i 1} = 1$, $p_{L1} = 0$ and $p_{G_j 1} = 0$ for $j \neq i$, then

$$U_L^C(0, 0, ..., 0, 1, 0, ..., 0) = \sum_{k=1}^{N-1} \binom{N}{k} \theta^{N-k} (1-\theta)^k \widetilde{U}(w, N+1-k),$$

$$U_{G_i}^C(0, 0, ..., 0, 1, 0, ..., 0) = \sum_{k=1}^{N} \binom{N}{k} \theta^{N-k} (1-\theta)^k \widetilde{U}(s, N+1-k),$$

$$U_{G_j}^C(0, 0, ..., 0, 1, 0, ..., 0) = \theta \Big[\sum_{k=1}^{N-1} \binom{N-1}{k} \theta^{N-k-1} (1-\theta)^k \widetilde{U}(s, N+1-k) \Big]$$

$$+(1-\theta)\Big[\sum_{k=2}^{N-1}\binom{N-1}{k}\theta^{N-k-1}(1-\theta)^k\widetilde{U}(w,N+1-k)\Big].$$

Proof S.IV *The proof follows the reasoning of the proof of Lemma 1 in the paper. The main point of the proof is to detail the number of strong and weak firms in all the different states. The complete proof is available upon request.* □

Once we have characterized firms' continuation payoff, we can derive firm's expected payoff in period 1. When firms face a public authority designing the first-period direct mechanism $\mathcal{M}_1$, their expected payoffs are equal to the sum of the first-period profits and the continuation payoff. Hence, a firm i, with production cost c_{i1} in period 1, has expected payoff given by

$$U_i(c_{i1}) = E_{c_{-i1}}[T_{i1}(c_1) - c_{i1}p_{i1}(c_1) + U_i^C(p_1(c_1))], \tag{S.11}$$

where $T_{i1}(c_1)$ and $p_{i1}(c_1)$ are, respectively, the payment and allocation rules in the first-period mechanism, such that $p_1(c_1) = (p_{i1}(c_1), p_{-i1}(c_1))$.

Replacing $U_i^C(.)$, defined in Lemma S.3, in equation (S.11), we obtain that the local firm's expected payoff in period 1 is given by

$$U_L(c_{L1}) = E_{c_{-L1}}\Big[T_{L1}(c_1) - c_{L1}p_{L1}(c_1) + p_{L1}U_L^C(1,0,...,0)$$
$$+(1-p_{L1}(c_1))U_L^C(0,0,...,0,1,0,...,0)\Big], \tag{S.12}$$

and global firm i's expected payoff is given by

$$U_G(c_{G_i1}) = E_{c_{-G_i1}}\Big[T_{G_i1}(c_1) - c_{G_i1}p_{G_i1}(c_1) + p_{G_i1}(c_1)U_{G_i}^C(0,0,...,0,1,0,...,0)$$
$$+(1-p_{G_i1}(c_1))[p_{L1}(c_1)U_{G_j}^C(1,0,...,0) + (1-p_{L1}(c_1))U_{G_j}^C(0,0,...,0,1,0,...,0)]\Big]. \tag{S.13}$$

Public Authority's continuation payoff: After transferability, before second-period costs are drawn. Program P_NI describes $W_2(X)$ the public authority's second-period payoff at state X. Applying results from Proposition S.1 in P_NI, we have that $W_2(X)$ is given by

$$W_2(X) = \int_{\Delta c_2(X)}\Big\{\sum_i\Big(S-(1+\lambda)c_{i2}-(1+\lambda-\alpha)\frac{F_{c_{i2}(c_{i2},X)}}{f_{c_{i2}(c_{i2},X)}}\Big)p_{i2}(c_2,X)\Big\}f_2(c_2\mid X)dc_2. \tag{S.14}$$

As $p_{i2}(c_2,X)$ corresponds to $v(h_i,N+1-k,c_i)$ in the notation where the number of strong and weak firms summarize the states, then the public authority's second-period payoff in the state that there are $N+1-k$ strong and k weak firms is

$$W_2(N+1-k) = (N+1-k)\Bigg\{ \int_{\Delta c_s} \Big[\Big(S-(1+\lambda)c-(1+\lambda-\alpha)\frac{F_s(c)}{f_s(c)}\Big)v(s,N+1-k,c)\Big] f_s(c)dc \Bigg\}$$

$$+k\Bigg\{ \int_{\Delta c_s} \Big[\Big(S-(1+\lambda)c-(1+\lambda-\alpha)\frac{F_w(c)}{f_w(c)}\Big)v(w,N+1-k,c)\Big] f_w(w)dc \Bigg\}. \tag{S.15}$$

After some algebraic manipulations (similar to the ones in the Proof of Lemma 2), we obtain that

$$W_2(N+1-k) = S_2(N+1-k) - (1+\lambda-\alpha)\sum_i \widetilde{U}_2^i(h_i, N+1-k). \tag{S.16}$$

where $S_2(N+1-k)$ is the expected net continuation consumers surplus consumers surplus minus expected payment to firms in the state where there are $N+1-k$ strong firms.

Note that $S_2(N+1-k)$ is defined as follows:

$$S_2(N+1-k) \equiv S - (1+\lambda)\Bigg\{ \int_{\Delta c_s} \Big[(N+1-k)v(s,N+1-k,c)c\Big] f_s(c)dc +$$

$$+ \int_{\Delta c_w} \Big[kv(w,N+1-k,c)c\Big] f_w(c)dc \Bigg\}. \tag{S.17}$$

Public Authority's continuation payoff: Before transferability. Having described public authority's payoff after transferability, let us turn to the characterization of firms' continuation payoff.

Definition 2 *We define*

$$g(x,N) = (N-x)\Big[\int_{\Delta c_s} cv(s,N-x,c)f_s(c)dc\Big] + (1+x)\Big[\int_{\Delta c_w} cv(w,N-x,c)f_w(c)dc\Big].$$

The following Lemma characterizes firms' continuation payoff.

Lemma S.4 *The expected net continuation consumers surplus $S^C(p_{L1}, p_{G_11}, ..., p_{G_N1})$ (consumers surplus net of expected payment to firms) and the public authority's continuation payoff $W^C(p_{L1}, p_{G_11}, ..., p_{G_N1})$ are such that*

(i) if the local firm is awarded the first-period public good provision, i.e. $p_{L1} = 1$ and

$p_{G_i1} = 0$ for all i, then

$$S^C(1,0,...,0) = S - (1+\lambda)\left\{\sum_{k=0}^{N}\binom{N}{k}\theta^{N-k}(1-\theta)^k g(k-1,N)\right\}, \tag{S.18}$$

$$W^C(1,0,...,0) = S^C(1,0,...,0) - (1+\lambda-\alpha)\Big(U_L^C(1,0,...,0) + \sum_{i=1}^{N} U_{G_i}^C(1,0,...,0)\Big); \tag{S.19}$$

(ii) if a global firm i is awarded the first-period public good provision, i.e. $p_{G_i1} = 1$, $p_{L1} = 0$ and $p_{G_j1} = 0$ for $j \neq i$, then

$$S^C(0,0,...,0,1,0,...,0) = S - (1+\lambda)\left\{\theta^N g(0,N) + \sum_{k=1}^{N-1}\binom{N-1}{k}\theta^{N-k}(1-\theta)^k g(k,N)\right.$$

$$\left. + \sum_{k=1}^{N-1}\binom{N-1}{k-1}\theta^{N-k}(1-\theta)^k g(k-1,N) + (1-\theta)^N g(N-1,N)\right\}; \tag{S.20}$$

$$W^C(0,0,...,0,1,0,...,0) = S^C(0,0,...,0,1,0,...,0)$$
$$-(1+\lambda-\alpha)\Big(U_L^C(0,0,...,0,1,0,...,0) + U_{G_i}^C(0,0,...,0,1,0,...,0)$$
$$+ \sum_{i\neq j}^{N} U_{G_j}^C(0,0,...,0,1,0,...,0)\Big). \tag{S.21}$$

Proof S.V *The proof follows the reasoning of the proof of Lemma 2 in the paper. The main point of the proof is to detail the number of strong and weak firms in all the different states. The complete proof is available upon request.* □

Having characterized the public authority's continuation payoff, we can derive its expected total social welfare in period 1. It is the sum of the first-period social welfare and its continuation payoff. As in Program $P_N I$, the first-period social welfare can be expressed by

$$W_1 = E_{c_1}\Big[\sum_i \Big\{\big(p_{i1}(c_1)\big)S + \alpha\big(T_{i1}(c_1) - c_{i1}p_{i1}(c_1)\big) - (1+\lambda)T_{i1}(c_1)\Big\}\Big]. \tag{S.22}$$

So, the total public authority's payoff in period 1 can be written as

$$W = W_1 + E_{c_1}\Big[W^C(p_1(c_1))\Big], \tag{S.23}$$

where the first term is the first-period social welfare defined in (S.22), and the second term is the public authority's continuation payoff defined in Lemma S.4.

S.1.2.2 Optimal first-period mechanism

We turn to the analysis of the mechanism design problem faced by the public authority in period 1. The authority designs a first-period direct mechanism $\mathcal{M}_1$ that solves the following program

$$\max_{p_1(c_1),T_1(c_1)} \int_{\Delta_1} \sum_{i=L,G_1,...,G_N} \Bigg\{ \Big\{ \big(p_{i1}(c_1)\big)S + \alpha\big(T_{i1}(c_1) - c_{i1}p_{i1}(c_1)\big) - (1+\lambda)T_{i1}(c_1) \Big\}$$

$$+S^C(p_{L1}, p_{G_1 1}, ..., p_{G_N 1}) - (1+\lambda-\alpha)U_i^C(p_1(c_1)) \Bigg\} f_1(c_1)dc_1 \qquad (P_N II)$$

subject to

1. individual rationality constraints in period 1:

$$U_i(c_{i1}) \geq 0, \forall c_{i1} \in \Delta_{i1}, \forall i; \qquad (IR_1)$$

2. incentive compatibility constraints in period 1:

$$U_{i1}(c_{i1}) = U_{i1}(c_{i1}, c_{i1}) \geq U_{i1}(\hat{c}_{i1}, c_{i1}), \forall i, \forall c_{i1}, \hat{c}_{i1} \in \Delta_{i1}, \qquad (IC_1)$$

   where $U_{i1}(\hat{c}_{i1}, c_{i1}) = E_{c_{-i1}}[T_{i1}(\hat{c}_{i1}, c_{-i1}) - c_{i1}p_{i1}(\hat{c}_{i1}, c_{-i1}) + U_i^C(p_1(\hat{c}_{i1}, c_{-i1}))]$;

3. possibility constraints in period 1:

$$p_{i1}(c_1) \geq 0 \text{ and } \sum_i p_{i1}(c_1) = 1, \forall c_1 \in \Delta_1, \forall i, \qquad (PC_1)$$

   where $U_i(c_{i1})$ and $U_i^C(.)$ are defined, respectively, in (S.11) and in Lemma S.3 for $i = \{L, G_1, ..., G_N\}$, $\Delta_{i1} = [\underline{c_w}, \overline{c_w}]$ and $\Delta_1 = [\underline{c_w}, \overline{c_w}]^{N+1}$, $f_1(c_1) = f_w(c_{i1})f_w(c_{-i1})$, since firms' cost are drawn from the *weak* distribution in period 1.

The Envelope Theorem applied to the maximization problem in (IC_1) with respect to $\hat{c}_{i1}$ yields

$$\frac{dU_i(c_{i1})}{dc_{i1}} = -E_{c_{-i1}}[p_{i1}(c_{i1}, c_{-i1})] = -Q_{i1}(c_{i1}). \qquad (S.24)$$

Equation (S.24) is a local incentive condition. As in Program $P_N I$, it is a necessary and sufficient condition if $Q_{i1}(c_{i1})$ is non increasing in c_{i1}.

From equation (S.24), $U_{i1}(c_{i1})$ is strictly decreasing in c_{i1}. So, the individual rationality constraint (IR_1) is satisfied if $U_{i1}(\overline{c_w}) \geq 0$. Integrating equation (S.24), we have that

$$U_{i1}(c_{i1}) = U_{i1}(\overline{c_w}) + \int_{c_{i1}}^{\overline{c_w}} Q_{i1}(s_{i1}) ds_{i1}.$$

Integrating by parts, we find that

$$\int_{\Delta c_1} U_i(c_{i1}) f_{c_1}(c_1) dc_1 = U_i(\bar{c}_w) + \int_{\Delta c_1} \frac{F_{c_{i1}}(c_{i1})}{f_{c_{i1}}(c_{i1})} p_{i1}(c_1) f_{c_1}(c_1) dc_1.$$

Replacing it in Program $P_N II$, the public authority's problem described above can be written as follows

$$\begin{aligned} \max_{p_1(c_1)} \quad & \int_{\Delta_1} \Bigg\{ \left[S + S^C(1,0,...,0) - (1+\lambda)c_{L1} - (1+\lambda-\alpha)\frac{F_w(c_{L1})}{f_w(c_{L1})} \right] p_{L1}(c_1) \\ & + \sum_{i=1}^{N} \left[S + S^C(0,0,...,0,1,0,...,0) - (1+\lambda)c_{G_i1} - (1+\lambda-\alpha)\frac{F_w(c_{Gi1})}{f_w(c_{G_i1})} \right] p_{G_i1}(c_1) \Bigg\} f_1(c_1) dc_1 \\ & -(1+\lambda-\alpha)\sum_i U_{i1}(\overline{c_w}), \qquad\qquad (P_N^{\prime} II) \end{aligned}$$

subject to

$U_{i1}(\overline{c_w}) \geq 0, \forall i;$

$\frac{dQ_{i1}(c_{i1})}{dc_{i1}} \leq 0, \forall i;$

$p_{i1}(c_1) \geq 0 \ \forall c_1 \in \Delta_1, \forall i$, and $\sum_i p_{i1}(c_1) = 1$.

Definition 3 *We define G_{*1} as the most efficient global firm in period 1, i.e. the one with the lowest revealed cost, such that $c_{G_*1} \equiv \min\{c_{G_i1}, \forall i \in \{1,...,N\}\}$.*

The following proposition characterizes the first-period optimal mechanism.

Proposition S.2 *The optimal first-period direct mechanism satisfies:*

(i) $U_{i1}(\overline{c_w}) = 0, \forall i;$

(ii) $p_{L1}(c_1) = 1$ *and* $p_{G_i}(c_1) = 0$ *if*

$$S^C(1,0,...,0) - \Phi_1(c_{L1}) \geq S^C(0,0,...,0,1,0,...,0) - \Phi_1(c_{G_*1}), \tag{S.25}$$

where $\Phi_1(c_i) = (1+\lambda)c_i + (1+\lambda-\alpha)\frac{F_w(c_i)}{f_w(c_i)}$ *is firm's first-period virtual cost; otherwise* $p_{L1}(c_1) = 0$ *and* $p_{G_*1}(c_1) = 1$.

Proof S.VI *The Proof is similar to the one of Proposition 3 in the paper.* □

Proposition S.3 *Consider the following condition:*

$$\Big[\int_{\Delta c_w} c[1-\hat{F}_s(\Phi_w(c_i))]^{N+1-k}[1-\hat{F}_w(\Phi_w(c_i))]^k f_w(c)dc\Big] \geq \Big[\int_{\Delta c_s} c[1-\hat{F}_s(\Phi_s(c_i))]^{N-k}[1-\hat{F}_w(\Phi_s(c_i))]^k f_s(c)dc\Big], \tag{S.26}$$

where $N+1-k$ is the number of strong firms and k is the number of weak firms.

Suppose that $\theta > 0$ and equation (S.26) holds. Then, the optimal discrimination policy in the first-period procurement mechanism in the city-economy with N global firms, is such that

(i) if the number of global firms N is finite, then for any N, and for any profile of revealed first-period costs $(c_{L1}, c_{G_1 1}, ..., c_{G_N 1})$, there exists $\bar{c}_{L1}(N) > c_{G_* 1}$ such that the local firm is selected to be the public good provider with probability one if $c_{L1} \leq \bar{c}_{L1}(N)$; otherwise the most efficient global firm G_{*1} is selected.

(ii) if N goes to infinite and the product $\theta \times N$ goes to a finite number, then for any profile of revealed costs $(c_{L1}, c_{G_1 1}, ..., c_{G_N 1})$, the local firm is selected to be the public good provider with probability one if $c_{L1} \leq c_{G_* 1}$, otherwise the most efficient global firm G_{*1} is selected.

The condition in equation (S.26) states that, for a given number N of firms, the expected cost of the second-period provider is decreasing in the number of *strong* firms in the second period, having the second-period contract awarded according to Proposition S.1. Note that, in the context of N global firms, condition (S.26) has the same interpretation as the condition in equation (14) in the paper.

Below we demonstrate Proposition S.3.

Proof S.VII *Define*

$$\Omega(c_{L1}, c_{G_* 1}, N) \equiv S^C(1,0,...,0) - \Phi_1(c_{L1}) - \big(S^C(0,0,...,0,1,0,...,0) - \Phi_1(c_{G_* 1})\big)$$

$$= \Phi_1(c_{G_* 1}) - \Phi_1(c_{L1}) + S^C(1,0,...,0) - S^C(0,0,...,0,1,0,...,0). \tag{S.27}$$

For any profile of revealed first-period costs $(c_{L1}, c_{G_1 1}, ..., c_{G_N 1})$, there exists $c_{G_* 1}$, such that, according to Proposition S.2, the local firm is selected in the period 1 if $\Omega(c_{L1}, c_{G_* 1}, N) \geq 0$.

Proof of Proposition S.3 (i). We have to show that for any finite N, there exists $\bar{c}_{L1}(N) > c_{G_* 1}$, such that $\Omega(\bar{c}_{L1}(N), c_{G_* 1}, N) \geq 0$. For this purpose, it is enough to show that

(A) $\frac{\partial \Omega(c_{L1}, c_{G_* 1}, N)}{\partial c_{L1}} < 0$;

(B) $\Omega(c_{G_* 1}, c_{G_* 1}, N) > 0$,

as by continuity of $\Omega(c_{L1}, c_{G_*1}, N)$, *(A) and (B) imply that there exists* $\bar{c}_{L1}(N) > c_{G_*1}$ *such that for any* $c_{L1} < \bar{c}_{L1}(N)$ *we have* $\Omega(\bar{c}_{L1}(N), c_{G_*1}, N) \geq 0$.

Before showing that (A) and (B) hold, note that $S^C(1, 0, ..., 0)$, *defined in Lemma S.4, can be written as follows*

$$S - (1+\lambda)\Bigg\{\theta^N g(-1, N) + \sum_{k=1}^{N-1} \binom{N-1}{k} \theta^{N-k}(1-\theta)^k g(k-1, N)$$

$$+ \sum_{k=1}^{N-1} \binom{N-1}{k-1} \theta^{N-k}(1-\theta)^k g(k-1, N) + (1-\theta)^N g(N-1, N)\Bigg\},$$

because

$$\binom{N}{k} - \binom{N-1}{k-1} = \binom{N-1}{k}.$$

Then, we replace $S^C(1, 0, ..., 0)$ *and* $S^C(0, 0, ..., 0, 1, 0, ..., 0)$, *defined in Lemma S.4, in (S.27) such that*

$$\Omega(c_{L1}, c_{G_*1}, N) = \Phi_1(c_{G_*1}) - \Phi_1(c_{L1})$$

$$+(1+\lambda)\Bigg\{\theta^N(g(0, N) - g(-1, N)) + \sum_{k=1}^{N-1} \binom{N-1}{k} \theta^{N-k}(1-\theta)^k (g(k, N) - g(k-1, N))$$

$$+ \sum_{k=1}^{N-1} \binom{N-1}{k-1} \theta^{N-k}(1-\theta)^k \underbrace{(g(k-1, N) - g(k-1, N))}_{0} + (1-\theta)^N \underbrace{(g(N-1, N) - g(N-1, N))}_{0}\Bigg\}.$$

Simplifying, we obtain

$$\Omega(c_{L1}, c_{G_*1}, N) = \Phi_1(c_{G_*1}) - \Phi_1(c_{L1})$$

$$+(1+\lambda)\Bigg\{\theta^N(g(0, N) - g(-1, N)) + \sum_{k=1}^{N-1} \binom{N-1}{k} \theta^{N-k}(1-\theta)^k (g(k, N) - g(k-1, N))\Bigg\}. \quad \text{(S.28)}$$

Equation (S.28) can be rewritten as

$$\Omega(c_{L1}, c_{G_*1}, N) = \Phi_1(c_{G_*1}) - \Phi_1(c_{L1})$$

$$+(1+\lambda)\theta\Bigg\{\sum_{k=0}^{N-1} \binom{N-1}{k} \theta^{N-k-1}(1-\theta)^k (g(k, N) - g(k-1, N))\Bigg\}. \quad \text{(S.29)}$$

Let us demonstrate that (A) is satisfied. Deriving expression (S.29) with respect to c_{L1}, *we have that*

$$\frac{\partial \Omega(c_{L1}, c_{G_*1}, N)}{\partial c_{L1}} = -\frac{\partial \Phi_1(c_{L1})}{\partial c_{L1}},$$

which is negative since we assume that $\frac{F_{it}(c)}{f_{it}(c)}$ *is nondecreasing in c. So, (A) holds.*

We now demonstrate that (B) holds. Evaluating $\Omega(c_{L1}, c_{G_*1}, N)$ *when* $c_{L1} = c_{G_*1}$*, we have that*

$$\Omega(c_{G_*1}, c_{G_*1}, N) = (1+\lambda)\theta \left\{ \sum_{k=0}^{N-1} \binom{N-1}{k} \theta^{N-k}(1-\theta)^k (g(k,N) - g(k-1,N)) \right\}$$

Note that to show that $\Omega(c_{G_*1}, c_{G_*1}, N) > 0$*, it is sufficient to show that* $g(x,N) - g(x-1,N) > 0$*. As it turns out, it is enough to show that* $\frac{\partial g(x,N)}{\partial x} > 0$.

From Definition 2, we have that

$$g(x,N) = (N-x)\Big[\int_{\Delta c_s} cv(s, N-x, c) f_s(c) dc\Big] + (1+x)\Big[\int_{\Delta c_w} cv(w, N-x, c) f_w(c) dc\Big]. \tag{S.30}$$

Deriving equation (S.30) with respect to x, we obtain that

$$\frac{\partial g(x,N)}{\partial x} = \left\{ \Big[\int_{\Delta c_w} cv(w, N-x, c) f_w(c) dc\Big] - \Big[\int_{\Delta c_s} cv(s, N-x, c) f_s(c) dc\Big] \right\} +$$

$$-\left\{ (N-x)\Big[\int_{\Delta c_s} c \frac{\partial v(s, N-x, c)}{\partial N} f_s(c) dc\Big] + (1+x)\Big[\int_{\Delta c_w} c \frac{\partial v(w, N-x, c)}{\partial N} f_w(c) dc\Big] \right\}.$$

Since $\frac{\partial v(s,N-x,c)}{\partial N} < 0$ *(see Lemma S.2 (III)), then it follows that* $g(x,N)$ *is strictly increasing in x if*

$$\Big[\int_{\Delta c_w} cv(w, N-x, c) f_w(c) dc\Big] \geq \Big[\int_{\Delta c_s} cv(s, N-x, c) f_s(c) dc\Big] \Bigg\}. \tag{S.31}$$

Based on the definition of $v(h, N-x, c)$ *for* $h = \{s, w\}$*, the expression (S.31) can be written as follows:*

$$\Big[\int_{\Delta c_w} c[1-\hat{F}_s(\Phi_w(c_i))]^{N+1-k}[1-\hat{F}_w(\Phi_w(c_i))]^k f_w(c) dc\Big] \geq \Big[\int_{\Delta c_s} c[1-\hat{F}_s(\Phi_s(c_i))]^{N-k}[1-\hat{F}_w(\Phi_s(c_i))]^k f_s(c) dc\Big],$$

which is the condition equation (S.26) in the statement of the Proposition S.3. The *expression in the left-hand side of the equation above is the expected cost of the second-period provider when there are $(N-k)$ strong and $(k+1)$ weak firms competing for the second-period contract. The expression in the right-hand side is the expected cost of the second-period provider when there are $(N-k+1)$ strong and k weak firms competing for the second-period contract.*

Consequently, $\Omega(c_{G_*1}, c_{G_*1}, N) > 0$, if equation (S.26) holds.

Because $\Omega(.,.,N)$ is decreasing in $c_{L1} \in (\underline{c}_w, \overline{c}_w)$, then, by continuity, there exists $\overline{c}_{L1} > c_{G_*1}$, such that the local firm is selected to be the public good provider with probability one when $c_{L1} \leq \overline{c}_{L1}$. The value $\overline{c}_{L1}$ depends on c_{G_*1}, since they jointly determine the value of the function $\Omega_1(.,.)$. In particular, note that if $\Omega(\overline{c}_w, c_{G_*1}, N) \geq 0$, then $\overline{c}_{L1} = \overline{c}_w$, which means that for a given revealed cost of the most efficient global firm's c_{G_*1}, and for any the local firm's revealed cost c_{L1}, then local firm is always selected. Yet in the case in which $\Omega(\overline{c}_w, c_{G_*1}, N) < 0$, then, by continuity and monotonicity of $\Omega(.,.,N)$ in c_{L1}, there exists $\overline{c}_{L1} \in (c_{G_*1}, \overline{c}_w)$, such that for any $c_{L1} \leq \overline{c}_{L1}$, then $\Omega(c_{L1}, c_{G_*1}, N) \geq 0$, so the local firm is selected to be the public good provider in period 1. Otherwise, for any $c_{L1} > \overline{c}_{L1}$, then $\Omega(c_{L1}, c_{G_*1}, N) < 0$, so the most efficient global one is selected. Note that in this case, $\overline{c}_{L1}$ is implicity defined by the expression $\Omega(\overline{c}_{L1}, c_{G_*1}, N) = 0$ which is an increasing function of c_{G_*1}.

Proof of Proposition S.3 (ii). It is enough to show that

$$\lim_{N \to +\infty} \Omega(c_{G_*1}, c_{G_*1}, N) = 0,$$

since this will imply that when N goes to infinite, $\Omega(c_{G_*1}, c_{G_*1}, N) \leq 0$ if and only if $c_{L1} < c_{G_*1}$. Let us first define $\Omega^* \equiv \frac{\Omega(c_{G_*1}, c_{G_*1}, N)}{(1+\lambda)\theta}$, $G(x,N) \equiv g(x,N) - g(x-1,N)$, and $m \equiv N-1$. Note that $\lim_{N \to +\infty} \Omega(c_{G_*1}, c_{G_*1}, N) = 0$ is equivalent to $\lim_{m \to +\infty} \Omega(c_{G_*1}, c_{G_*1}, m+1) = 0$.

We have that

$$\Omega^* = \sum_{k=0}^{m} \binom{m}{k} \theta^{m-k} (1-\theta)^k G(k, m+1). \tag{S.32}$$

As it turns out, $G(x, m+1)$ is a function of the random variable x that is distributed according to a Binomial $B(m,p)$. Therefore, $\mu_m(k) \equiv \binom{m}{k}\theta^{m-k}(1-\theta)^k$ is the probability that $x = k$. Thus, we can write equation (S.32) as

$$\Omega^* = \sum_{k=0}^{m} \mu_m(k) G(k, m+1) = E[G(x, m+1)].$$

So, Ω^* is the expected value of $G(x, m+1)$ when x is distributed according to a Binomial $B(m, p)$.

If we assume that m goes to infinite and that the product $\theta \times m$ goes to a finite number κ, then the Binomial $B(m, p)$ converges in distribution to a Poisson distribution Pois(κ). Consequently, the expected value of x goes to κ. Because $G(x, m + 1)$ is a continuous function of x, then the expected value of $G(x, m+1)$ goes to the expected value of $G(\kappa, m+1)$. Let us now show that $G(\kappa, m + 1)$ goes to 0 as m goes to infinite.
By definition $G(\kappa, m + 1) = g(\kappa, m + 1) - g(\kappa - 1, m + 1)$ can be written as

$$G(\kappa, m+1) = (m-\kappa+1)\Big[\int_{\Delta c_s} cv(s, m-\kappa+1, c) f_s(c) dc\Big] + (\kappa+1)\Big[\int_{\Delta c_w} cv(w, m-\kappa+1, c) f_w(c) dc\Big]$$
$$-\Big[(m-\kappa+2)\Big[\int_{\Delta c_s} cv(s, m-\kappa+2, c) f_s(c) dc\Big] + \kappa\Big[\int_{\Delta c_w} cv(w, m-\kappa+2, c) f_w(c) dc\Big]\Big]. \quad \text{(S.33)}$$

In order to find the limit, we find an upper bound for $G(\kappa, m+1)$ denoted by $\bar{G}(\kappa, m+1)$ and a lower bound denoted by $\underline{G}(\kappa, m + 1)$ such that

$$\bar{G}(\kappa, m+1) \equiv (m-\kappa+1)\Big[\int_{\Delta c_s} cv(s, m-\kappa+1, c) f_s(c) dc\Big] + (\kappa+1)\Big[\int_{\Delta c_w} cv(w, m-\kappa+1, c) f_w(c) dc\Big]$$
$$-\Big[(m-\kappa+2)\Big[\int_{\Delta c_s} cv(s, m-\kappa+1, c) f_s(c) dc\Big] + \kappa\Big[\int_{\Delta c_w} cv(w, m-\kappa+1, c) f_w(c) dc\Big]\Big].$$

Note that,

$$G(\kappa, m + 1) \leq \bar{G}(\kappa, m + 1),$$

as $v(h, y, c)$ is decreasing in y, $\forall i$,$\forall h$, by Lemma S.2 (III). Similarly,

$$\underline{G}(\kappa, m+1) \equiv (m-\kappa+1)\Big[\int_{\Delta c_s} cv(s, m-\kappa+2, c) f_s(c) dc\Big] + (\kappa+1)\Big[\int_{\Delta c_w} cv(w, m-\kappa+2, c) f_w(c) dc\Big]$$
$$-\Big[(m-\kappa+2)\Big[\int_{\Delta c_s} cv(s, m-\kappa+1, c) f_s(c) dc\Big] + \kappa\Big[\int_{\Delta c_w} cv(w, m-\kappa+1, c) f_w(c) dc\Big]\Big].$$

Note that,

$$\underline{G}(\kappa, m + 1) \leq G(\kappa, m + 1),$$

as $v(h, y, c)$ is decreasing in y, $\forall i$,$\forall h$, by Lemma S.2 (III).

Taking into account that $\lim_{m\to+\infty} v(h, m - \kappa + 1, c) = 0, \forall h$, from Lemma S.2 (II), we have that

$$\lim_{m\to+\infty} \bar{G}(\kappa, m + 1) = \lim_{m\to+\infty} \underline{G}(\kappa, m + 1) = 0,$$

and then $G(\kappa, m+1) \to 0$ *as* $m \to +\infty$.

$\square$

Proposition S.3 shows that, under the condition in equation (S.26), if the number of global firms N is finite, then it is optimal to bias the first-period procurement in favor of the local firm. Nevertheless, as the number of global firms becomes very large (i.e. N goes to infinite) and when the number of markets where global firms can be incumbent remains constant such that the chance of a global firm to have transferability is small when many firms are present (i.e. $\theta \times N$ goes to a finite number when N goes to infinite), the first-period procurement is awarded to the firm with the lowest cost for any profile of revealed costs.

S.2 Optimal Procurement Mechanism with Commitment

In this section we consider the optimal procurement design with learning-by-doing when the public authority can commit to a long-term mechanism. We examine the case that the public authority commits to a two-period contract at the beginning of period 1 after the firms are privately informed of the first-period costs.

Mechanism Design. The authority designs a procurement mechanism to select and to pay firms for the public good provision for a two-period contract at the beginning of period 1 after the firms are privately informed of the first-period costs. It maximizes its expected social welfare subject to the constraints imposed by its lack of knowledge about firms' costs. As the authority and the firms are uninformed of the second-period costs when the mechanism is proposed, we can apply the standard Revelation Principle proposed by Myerson (1981). By the Revelation Principle, for any optimal mechanism there is an equivalent direct mechanism in which firms reveals their private first-period production cost, and the project is awarded and payments are made according to the costs revealed. The *optimal direct mechanism* is then defined as $\mathcal{M} = \{\Delta, p(c_1), T(c_1)\}$, where $\Delta = (\Delta_L, \Delta_G)$ is the set of possible costs for each firm in period 1; $c_1 = (c_{L1}, c_{G1})$ is the vector of true costs in period 1; $p(c_1) = (p_L(c_1), p_G(c_1))$ is the vector of the probability of awarding the project for two periods to each firm; $T = (T_L, T_G)$ is the vector of expected payment to firms.

The direct mechanism $\mathcal{M}$ maximizes the social welfare, subject to three constraints: individual rationality constraints, incentive compatibility constraints, and possibility constraints.

Payoffs. The expected profit of firm i in period 1 is denoted by

$$U_i(c_{i1}) = E_{c_{-i1}}\Big[T_i(c_1) - p_{i1}(c_1)\big[c_{i1} + \int_{\underline{c}_s}^{\overline{c}_s} c f_s(c) dc\big]\Big], \tag{S.34}$$

where $T_i(c_1)$ is the monetary transfer that firm i receives for the public good provision for two periods, c_{i1} is its production cost in period 1 and $\int_{\underline{c}_s}^{\overline{c}_s} c f_s(c) dc$ is its expected production cost in period 2, with $p_i(c_1)$ the firm i's probability of being the public good provider. Note that $E_s[c] = \int_{\underline{c}_s}^{\overline{c}_s} c f_s(c) dc$.

Hence, the public authority's objective function is described as follows:

$$W = \int_\Delta \Big\{\big(\sum_i p_i(c_1)\big)2S + \alpha \sum_i \big[T_i(c_1) - p_i(c_1)(c_{i1} + E_s[c])\big]$$

$$-(1+\lambda)\big(\sum_i T_i(c_1)\big)\Big\} f_1(c_1) dc_1, \tag{S.35}$$

with $\Delta = \Delta_w \times \Delta_w$, and $f_1(c_1) = f_w(c_{i1}) f_w(c_{-i1})$.

Optimal Mechanism. The public authority designs $\mathcal{M}$ that solves

$$\max_{p(c_1), T(c_1)} W \qquad \text{subject to} \tag{P_C}$$

1. individual rationality constraints:

$$U_i(c_{i1}) \geq 0, \forall i, \forall c_{i1} \in \Delta_w; \tag{IR_C}$$

2. incentive compatibility constraints:

$$U_i(c_{i1}) = U_{i1}(c_{i1}, c_{i1}) \geq U_i(\hat{c}_{i1}, c_{i1}),\ \forall i, \forall c_{i1}, \hat{c}_{i1} \in \Delta_w, \tag{IC_C}$$

with $U_{i1}(\hat{c}_{i1}, c_{i1}) = E_{c_{-i1}}\big[T_{i1}(\hat{c}_{i1}, c_{-i1}) - p_i(\hat{c}_{i1}, c_{-i1})(c_{i1} + E_s[c])\big]$;

3. possibility constraints:

$$p_i(c_1) \geq 0, \forall i,\ \text{and}\ \sum_i p_i(c_1) = 1, \forall c_1 \in \Delta. \tag{PC_C}$$

We apply the Envelope Theorem to firms' maximization problem in (IC_C) with respect to

$\hat{c}_{i1}$ which yields

$$\frac{dU_i(c_{i1})}{dc_{i1}} = -E_{c_{-i1}}[p_{i1}(c_{i1}, c_{-i1})] = -Q_i(c_{i1}). \tag{S.36}$$

Equation (S.36) is a local incentive condition. It is a necessary and sufficient condition if the following condition holds:

$$\frac{dQ_i(c_{i1})}{dc_{i1}} \leq 0.$$

From equation (S.36), $U_i(c_{i1})$ is strictly decreasing in c_i. So the individual rationality constraint (IR_C) is satisfied if $U_i(\overline{c_i}) \geq 0$. Integrating (S.36), we have that

$$U_i(c_{i1}) = U_{i1}(\overline{c_{i1}}) + \int_{c_{i1}}^{\overline{c_{i1}}} Q_i(s_{i1}) ds_{i1}. \tag{S.37}$$

Thus, the public authority's problem P_C can be rewritten as:

$$\max_{p(c_1)} \int_\Delta \Bigg\{ \left[2S - (1+\lambda)(c_{L1} + E_s[c]) - (1+\lambda-\alpha)\frac{F_w(c_{L1})}{f_w(c_w)} \right] p_L(c_1)$$
$$+ \left[2S - (1+\lambda)(c_{G2} + E_s[c]) - (1+\lambda-\alpha)\frac{F_w(c_{G1})}{f_w(c_{G1})} \right] p_G(c_1) \Bigg\} f_w(c_{L1}) f_w(c_{G1}) dc_{L1} dc_{G1}$$
$$-(1+\lambda-\alpha)\Big\{ U_L(\overline{c_w}) + U_G(\overline{c_w}) \Big\} \tag{S.38}$$

subject to

$$U_i(\overline{c_w}) \geq 0, \forall i;$$
$$\frac{dQ_{i1}(c_{i1})}{dc_{i1}} \leq 0, \forall i;$$
$$p_i(c_1) \geq 0, \forall c_1 \in \Delta_w \times \Delta_w, \forall i, \text{ and } \sum_i p_i(c_1) = 1.$$

The optimal second-period mechanism is the solution of the pointwise maximization problem above. The Proposition S.4 characterizes the optimal mechanism.[2]

Proposition S.4 *The optimal mechanism satisfies:*

(i) $U_i(\overline{c_w}) = 0, \forall i$;

(ii) $p_L(c_1) = 1$ *and* $p_G(c_1) = 0$ *if*

$$c_{L1} \leq c_{G1}, \tag{S.39}$$

[2]The proof of Proposition S.4 is omitted as it is similar to the results presented in Myerson (1981), McAfee and McMillan (1989), and Naegelen and Mougeot (1998).

otherwise $p_L(c_1) = 0$ *and* $p_G(c_1) = 1$.

Proposition S.4 shows that the public provision is awarded for two consecutive periods to the firm with the lowest cost in period 1. Hence, when the public authority can commit to a long-term contract there is no handicapping in period 1.

S.3 Optimal Dynamic Procurement Mechanism: Global Firm with Synergy Disadvantages

In this section we characterize the optimal dynamic procurement mechanism when a global firm can become weak due to commitment of some vital resources to other markets. Precisely, we examine the case in which the global firm becomes weak with probability θ. Note that this alternative assumption does not affect the optimal second-period mechanism described in Proposition 1. However, it does change the continuation payoffs and the optimal first-period mechanism, as we describe below.

S.3.1 First-period Procurement

In order to characterize firms' first-period strategy and public authority's problem in period 1, we first compute the continuation payoffs of the firms and the public authority.

S.3.1.1 Continuation payoffs

As in Section 5 in the paper, the continuation payoffs are computed at the end of the first period, after first-period public good provision was awarded and before Nature draws the transferability. In period 1, neither the authority nor the firms know firms' second-period costs and global firm's transferability. However, as the public authority will optimally select and pay firms in period 2 according to the second-period mechanism described in Proposition 1, we can compute firms' expected equilibrium payoff and public authority's expected equilibrium payoff at the beginning of period 2.

Firms' continuation payoff. Let $\underset{\sim}{U} \equiv \int_{\underline{c}_w}^{\bar{c}_w} (1 - F_w(c))F_w(c)dc$ be the second-period expected payoff of a weak firm when it faces a weak opponent.

Lemma S.5 *The continuation payoffs of the firms are such that*

(i) if the local firm is awarded the first-period public good provision, i.e. $p_{L1} = 1$ *and* $p_{G1} = 0$*, then* $U_L^C(1,0) = \overline{U}$ *and* $U_G^C(1,0) = \underline{U}$*;*

(ii) if the global firm is awarded the first-period public good provision, i.e. $p_{L1} = 0$ *and* $p_{G1} = 1$*, then* $U_L^C(0,1) = \theta \underset{\sim}{U} + (1-\theta)\underline{U}$ *and* $U_G^C(0,1) = \theta \underset{\sim}{U} + (1-\theta)\overline{U}$*.*

Proof of Lemma S.5. The proof is omitted as it is similar to the proof of Lemma 1.

When that the local firm is selected in period 1, there will be one strong firm (incumbent local) and one weak firm (entrant global) competing for public good provision in period 2. Yet, when the global firm is selected in period 1, with probability θ, the global firm becomes weak due to commitment of some vital resources to other markets, even though it is an incumbent in the city-economy. In this case, there will be two weak firms (i.e. incumbent global in other markets, and entrant local) competing for the public good provision in period 2. With probability $1-\theta$, the global firm does not become weak as it is not in other markets (there will be no resources committed). Then, in this case, there will be one strong firm (incumbent global) and one weak firm (entrant firm) competing for public good provision in period 2.

Public Authority's continuation payoff. As in Section 5 in the paper, we denote by $W^C(p_{L1}, p_{G1})$ and $S^C(p_{L1}, p_{G1})$, respectively, the public authority's continuation payoff and expected net continuation consumers' surplus (consumers surplus minus expected payment to firms). We define by

$$\underset{\sim}{S} \equiv S - 2(1+\lambda)E_w\big[E_w[c \cdot 1\{c \leq c''\}|c'']\big], \tag{S.40}$$

$$\underset{\sim}{W} \equiv \underset{\sim}{S} - (1+\lambda-\alpha)2\underset{\sim}{U}, \tag{S.41}$$

where $1\{\cdot\}$ is an indicator function, c and c'' according to $F_w(.)$.

The term $\underset{\sim}{S}$ represents the net expected consumers' surplus when two weak firms are competing in the second period. Similarly, $\underset{\sim}{W}$ represents the expected welfare derived by the authority when two weak firms are competing in the second period.

The following Lemma characterizes the public authority's continuation payoff.

Lemma S.6 *The public authority's continuation payoff is such that*

(i) if the local firm is awarded the first-period public good provision, i.e. $p_{L1} = 1$ *and* $p_{G1} = 0$*, then*

$$W^C(1,0) = \underline{W} \qquad \text{and} \qquad S^C(1,0) = \underline{S}; \tag{S.42}$$

(ii) if the global firm is awarded the first-period public good provision, i.e. $p_{L1} = 0$ *and* $p_{G1} = 1$*, then*

$$W^C(0,1) = \theta \underset{\sim}{W} + (1-\theta)\underline{W} \qquad \text{and} \qquad S^C(0,1) = \theta \underset{\sim}{S} + (1-\theta)\underline{S}. \tag{S.43}$$

Note that $\underline{W}$ and $\underline{S}$ are defined in Section 5 in the paper.

Proof of Lemma S.6. The proof is omitted as it is similar to the proof of Lemma 2.

The following proposition compares the public authority's continuation payoff $W^C(.)$ and expected net continuation surplus $S^C(.)$ in the two cases described in Lemma S.6.

Proposition S.5 *The public authority's continuation payoff and expected net consumers surplus functions are such that:*

(i) $\underset{\sim}{W} < \underline{W}$*, which implies that* $W^C(1,0) > W^C(0,1)$*;*

(ii) $\underset{\sim}{S} \leq \underline{S}$ *if and only if*

$$\int_{\underline{c}_w}^{\overline{c}_w} \Big[\int_{\underline{c}_w}^{c} \widetilde{c} f_w(\widetilde{c}) d\widetilde{c} \Big] f_w(c) dc \;\; \geq \;\; \frac{1}{2} \Bigg\{ \int_{\underline{c}_s}^{\overline{c}_s} \Big[\int_{\underline{c}_w}^{\Phi_w^{-1}(\Phi_s(c))} \widetilde{c} f_w(\widetilde{c}) d\widetilde{c} \Big] f_s(c) dc + + \int_{\underline{c}_w}^{\overline{c}_w} \Big[\int_{\underline{c}_s}^{\Phi_s^{-1}(\Phi_w(c))} \widetilde{c} f_s(\widetilde{c}) d\widetilde{c} \Big] f_w(c) dc \Bigg\}, \quad \text{(S.44)}$$

which implies that $S^C(1,0) > S^C(0,1)$*.*

Proof of Proposition S.5. The proof is similar to the one of Proposition 2.

The condition in equation (S.44) states that the expected cost of the second-period provider is lower when there are a strong and a weak firm competing for the second-period contract than when there are two weak firms competing for the same contract, having the second-period contract awarded according to the optimal mechanism described in Proposition 1. When equation (S.44) holds, Proposition S.5 shows that the expected net continuation consumers' surplus is strictly higher when the first-period provider is the local firm.

The intuition behind Proposition S.5 is the following. When the global firm is selected in period 1 (Lemma S.6 (i)), with probability θ there will be a competition between two weak (local and global) firms. This leads to *high* expected transfers (i.e., *low* consumer surplus) in the second period as both possible suppliers are going to be inefficient (i.e., high cost) in period 2. With probability $1 - \theta$, there will be mild competition between one strong (global) firm and one weak (local) firm. This may lead, when equation (S.44) holds, to relatively *low* expected transfer (i.e., *high* consumer surplus) in the second period, as at least one of possible suppliers is going to be efficient (i.e., low cost) in period 2. On the contrary, when the local firm is selected in period 1 (Lemma 2 (ii)), with probability one, one of the possible suppliers (the local firm) is going to be efficient (i.e., low cost) in period 2. This leads to relatively *low* expected transfer (i.e., *high* consumer surplus) in the second period. Note that intuition behind Proposition S.5 is different from the one on Proposition 2 in the paper.

Note that the reference to *low* expected transfer – associated with a competition between a strong and a weak firm – and the expression *high* expected transfer – associated with a competition between two weak firms – rely on the assumption that the expected cost of the provider in the second period is lower for the case of a strong and a weak firm than for the case of two weak firms competing for the second-period contract, which is the condition stated in equation (S.44).

Having characterized the public authority's continuation payoff, we can derive its expected intertemporal social welfare. It can be written as:

$$\underset{\sim}{W} = \int_{\Delta_1} \Big\{ W_1(p_1(c_1), T_1(c_1)) + p_{L1}(c_1)\underline{W} + (1 - p_{L1}(c_1))[\theta \underset{\sim}{W} + (1-\theta)\underline{W}] \Big\} f_1(c_1) dc, \quad \text{(S.45)}$$

where $f_1(c_1) = f_w(c_{L1}) f_w(c_{G1})$. The vector-functions $T_1(.) = (T_{L1}(.), T_{G1}(.))$ and $p_1(.) = (p_{L1}(.), p_{G1}(.))$ are respectively the first-period expected payments and allocation rule.

S.3.1.2 Optimal first-period mechanism

The authority designs the first-period direct mechanism $\mathcal{M}_1$ that solves a problem which is similar to the problem P_{II} in Section 5.2 in the paper. The difference is that now the public authority maximizes the intertemporal social welfare function defined in equation (S.45).

The following proposition characterizes the first-period optimal mechanism when a global firm can become weak due to commitment of some vital resources to other markets.

Proposition S.6 *The optimal first-period mechanism satisfies:*

(i) $U_{i1}(\overline{c_w}) = 0, \forall i$*;*

(ii) $p_{L1}(c_1) = 1$ *and* $p_{G1}(c_1) = 0$ *if*

$$\underline{S} - \Phi_1(c_{L1}) \geq \theta \underset{\sim}{S} + (1-\theta)\underline{S} - \Phi_1(c_{G1}), \quad \text{(S.46)}$$

where $\Phi_1(c_{i1}) = (1+\lambda)c_{i1} + (1+\lambda-\alpha)\frac{F_w(c_{i1})}{f_w(c_{i1})}$ *is firm i's first-period virtual cost; otherwise* $p_{L1}(c_1) = 0$ *and* $p_{G1}(c_1) = 1$.

Proof of Proposition S.6. The proof is similar to the one of Proposition 3.

From equation (S.46), the public authority awards the first-period public good provision to the firm with the highest net expected continuation consumers surplus $S^C(.)$ minus first-period virtual cost $\Phi_1(c_{i1})$. We can rewrite equation (S.46) as

$$\theta(\underline{S} - \underset{\sim}{S}) + \Phi_1(c_{G1}) \geq \Phi_1(c_{L1}),$$

where $\theta(\underline{S} - \underset{\sim}{S})$ represents the optimal bias.

We can show that it can be optimal for the public authority to discriminate in favor of the local firm in the first period. That means that the local firm may be optimally selected, even though it has higher first-period production cost than the global one.[3] By selecting a local firm in the first-period, the public authority can make sure that one of the possible suppliers (the local firm) is going to be efficient (i.e., low cost) in period 2. This leads to relatively *low* expected transfer in the second period. On the contrary, by selecting a global firm in period 1, there is a positive chance (probability θ) that both possible suppliers are going to be inefficient (i.e., high cost) in period 2. This leads to *high* expected transfers and *low* expected social welfare in the second period as both possible suppliers are going to be inefficient (i.e., high cost) in period 2.

S.4 The Optimal Second-Period Mechanism

This section describes the solution of the problem P_I in the paper. We start by analyzing $(IC_2(X))$, a constraint in problem P_I.

We apply the Envelope Theorem to firms' maximization problem in $(IC_2(X))$ with respect to $\hat{c}_{i2}$ which yields

$$\frac{dU_{i2}(c_{i2}, X)}{dc_{i2}} = -E_{c_{-i2}}[p_{i2}(c_{i2}, c_{-i2}, X)|X] = -Q_{i2}(c_{i2}, X). \tag{S.47}$$

Equation (S.47) is a local incentive condition. It is a necessary and sufficient condition if the following condition holds:

$$\frac{dQ_{i2}(c_{i2}, X)}{dc_{i2}} \leq 0.$$

From equation (S.47), $U_{i2}(c_{i2}, X)$ is strictly decreasing in c_{i2}. So the individual rationality constraint $(IR_2(X))$ is satisfied if $U_{i2}(\overline{c_{i2}}(X), X) \geq 0$. Integrating (S.47), we have that

$$U_{i2}(c_{i2}, X) = U_{i2}(\overline{c_{i2}}(X), X) + \int_{c_{i2}}^{\overline{c_{i2}}(X)} Q_{i2}(s_{i2}, X) ds_{i2}. \tag{S.48}$$

Hence, following Myerson (1981), McAfee and McMillan (1989), and Naegelen and Mougeot (1998), the public authority's problem P_I in period 2 can be rewritten as:

[3]The proof of this result is similar to the one in Proposition 4 in the paper.

$$\max_{p_2(c_2,X)} \quad \int_{\Delta_2(X)} \left\{ \left[S - (1+\lambda)c_{L2} - (1+\lambda-\alpha)\frac{F_{L2}(c_{L2}|X)}{f_{L2}(c_{L2}|X)} \right] p_{L2}(c_2, X) \right.$$
$$\left. + \left[S - (1+\lambda)c_{G2} - (1+\lambda-\alpha)\frac{F_{G2}(c_{G2}|X)}{f_{G2}(c_{G2}|X)} \right] p_{G2}(c_2, X) \right\} f_2(c_2|X) dc_2$$
$$-(1+\lambda-\alpha)\left\{ U_{L2}(\overline{c_{L2}}(X), X) + U_{G2}(\overline{c_{G2}}(X), X) \right\} \tag{S.49}$$

subject to

$$U_{i2}(\overline{c_{i2}}(X), X) \geq 0, \forall i;$$
$$\frac{dQ_{i2}(c_{i2}, X)}{dc_{i2}} \leq 0, \forall i;$$
$$p_{i2}(c_2, X) \geq 0, \forall c_2 \in \Delta_2(X), \forall i, \text{ and } \sum_i p_{i2}(c_2, X) = 1.$$

The optimal second-period mechanism is the solution of the pointwise maximization problem above. The Proposition 1 in the paper characterizes the optimal mechanism.[4]

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[4]The proof of Proposition 1 is omitted as it is similar to the results presented in Myerson (1981), McAfee and McMillan (1989), and Naegelen and Mougeot (1998).